

Transportation models: Special cases

Introduction

If the basic feasible solution of a transportation problem with m origins and n destinations has fewer than $m + n - 1$ positive x_{ij} (occupied cells), the problem is said to be a degenerate transportation problem.

Degeneracy can occur at two stages:

1. At the initial solution
2. During the testing of the optimal solution

To resolve degeneracy, we make use of an artificial quantity (d). The quantity d is assigned to that unoccupied cell, which has the minimum transportation cost.

The use of d is illustrated in the following example.

Example

Factory	Dealer				Supply
	1	2	3	4	
A	2	2	2	4	1000
B	4	6	4	3	700
C	3	2	1	0	900
Requirement	900	800	500	400	

Solution.

An initial basic feasible solution is obtained by Matrix Minimum Method.

Table 1

Factory	Dealer				Supply
	1	2	3	4	
A	2	2	2	4	1000
B	4	6	4	3	700
C	3	2	1	0	900
Requirement	900	800	500	400	

Number of basic variables = $m + n - 1 = 3 + 4 - 1 = 6$

Since number of basic variables is less than 6, therefore, it is a degenerate transportation problem.

To resolve degeneracy, we make use of an artificial quantity(d). The quantity d is assigned to that unoccupied cell, which has the minimum transportation cost.

In the above table, there is a tie in selecting the smallest unoccupied cell. In this situation, you can choose any cell arbitrarily. We select the cell C2 as shown in the following table.

Table 2

Factory	Dealer				Supply
	1	2	3	4	
A	2 ⁹⁰⁰	2 ¹⁰⁰	2	4	1000
B	4	6 ⁷⁰⁰	4	3	700
C	3	2 ^d	1 ⁵⁰⁰	0 ⁴⁰⁰	900 + d
Requirement	900	800 + d	500	400	2600 + d

Now, we use the stepping stone method to find an optimal solution.

Calculating opportunity cost

Unoccupied cells	Increase in cost per unit of reallocation	Remarks
A3	$+2 - 2 + 2 - 1 = 1$	Cost Increases
A4	$+4 - 2 + 2 - 0 = 4$	Cost Increases
B1	$+4 - 6 + 2 - 2 = -2$	Cost Decreases
B3	$+4 - 6 + 2 - 1 = -1$	Cost Decreases
B4	$+3 - 6 + 2 - 0 = -1$	Cost Decreases
C1	$+3 - 2 + 2 - 2 = 1$	Cost Increases

The cell B1 is having the maximum improvement potential, which is equal to -2. The maximum amount that can be allocated to B1 is 700 and this will make the current basic variable corresponding to cell B2 non basic. The improved solution is shown in the following table.

Table 3

Factory	Dealer				Supply
	1	2	3	4	
A	2 ²⁰⁰	2 ⁸⁰⁰	2	4	1000
B	4 ⁷⁰⁰	6	4	3	700
C	3	2 ^d	1 ⁵⁰⁰	0 ⁴⁰⁰	900
Requirement	900	800	500	400	2600

The optimal solution is
 $2 \times 200 + 2 \times 800 + 4 \times 700 + 2 \times d + 1 \times 500 + 0 \times 400 = 5300 + 2d.$

Notice that d is a very small quantity so it can be neglected in the optimal solution. Thus, the net transportation cost is 5300.

Unbalanced Transportation Problem

So far we have assumed that the total supply at the origins is equal to the total requirement at the destinations. But in certain situations, the total supply is not equal to the total demand. Thus, the transportation problem with unequal supply and demand is said to be unbalanced transportation problem.

If the total supply is more than the total demand, we introduce an additional column, which will indicate the surplus supply with transportation cost zero. Similarly, if the total demand is more than the total supply, an additional row is introduced in the table, which represents unsatisfied demand with transportation cost zero. The balancing of an unbalanced transportation problem is illustrated in the following example.

Example

Plant	Warehouse			Supply
	W1	W2	W3	
A	28	17	26	500
B	19	12	16	300
Demand	250	250	500	

Solution:

The total demand is 1000, whereas the total supply is 800.
Total supply < total demand.

To solve the problem, we introduce an additional row with transportation cost zero indicating the unsatisfied demand.

Plant	Warehouse			Supply
	W1	W2	W3	
A	28	17	26	500
B	19	12	16	300
Unsatisfied demand	0	0	0	200
Demand	250	250	500	1000

Using matrix minimum method, we get the following allocations.

Plant	Warehouse			Supply
	W1	W2	W3	
A	50 28	17	450 26	500
B	19	250 12	50 16	300
Unsatisfied demand	200 0	0	0	200
Demand	250	250	500	1000

Initial basic feasible solution

$$50 \times 28 + 450 \times 26 + 250 \times 12 + 50 \times 16 + 200 \times 0 = 16900.$$

Maximization In A Transportation

There are certain types of transportation problems where the objective function is to be maximized instead of being minimized. These problems can be solved by converting the maximization problem into a minimization problem.

Example

Surya Roshni Ltd. has three factories - X, Y, and Z. It supplies goods to four dealers spread all over the country. The production capacities of these factories are 200, 500 and 300 per month respectively.

Factory	Dealer				Capacity
	A	B	C	D	
X	12	18	6	25	200
Y	8	7	10	18	500
Z	14	3	11	20	300
Demand	180	320	100	400	

Determine a suitable allocation to maximize the total net return.

Solution.

Maximization transportation problem can be converted into minimization transportation problem by subtracting each transportation cost from maximum transportation cost. Here, the maximum transportation cost is 25. So subtract each value from 25. The revised transportation problem is shown below.

Table 1

Factory	Dealer				Capacity
	A	B	C	D	
X	13	7	19	0	200
Y	17	18	15	7	500
Z	11	22	14	5	300
Demand	180	320	100	400	

An initial basic feasible solution is obtained by matrix-minimum method and is shown in the final table.

Final table

Factory	Dealer				Capacity
	A	B	C	D	
X	13	7	19	0 ²⁰⁰	200
Y	17 ⁸⁰	18 ³²⁰	15 ¹⁰⁰	7	500
Z	11 ¹⁰⁰	22	14	5 ²⁰⁰	300
Demand	180	320	400	400	

The maximum net return is:

$$25 \times 200 + 8 \times 80 + 7 \times 320 + 10 \times 100 + 14 \times 100 + 20 \times 200 = 14280.$$

Prohibited Routes

Sometimes there may be situations, where it is not possible to use certain routes in a transportation problem. For example, road construction, bad road conditions, strike, unexpected floods, local traffic rules, etc. We can handle such type of problems in different ways:

- ← A very large cost represented by M or ∞ is assigned to each of such routes, which are not available.
- ← To block the allocation to a cell with a prohibited route, we can cross out that cell.

The problem can then be solved in its usual way

Example

Consider the following transportation problem.

Factory	Warehouse			Supply
	W ₁	W ₂	W ₃	
F ₁	16	∞	12	200
F ₂	14	8	18	160
F ₃	26	∞	16	90
Demand	180	120	150	450

Solution.

An initial solution is obtained by the matrix minimum method and is shown in the final table.

Final Table

Factory	Warehouse			Supply
	W ₁	W ₂	W ₃	
F ₁	16 ⁵⁰	∞	12 ¹⁵⁰	200
F ₂	14 ⁴⁰	8 ¹²⁰	18	160
F ₃	26 ⁹⁰	∞	16	90
Demand	180	120	150	450

Initial basic feasible solution

$$16 \times 50 + 12 \times 150 + 14 \times 40 + 8 \times 120 + 26 \times 90 = 6460.$$

The minimum transportation cost is 6460.

Time Minimizing Problem

Succinctly, it is a transportation problem in which the objective is to minimize the time. This problem is same as the transportation problem of minimizing the cost, expect that the unit transportation cost is replaced by the time t_{ij} .

Steps

- Determine an initial basic feasible solution using any one of the following:
 - ← North West Corner Rule
 - ← Matrix Minimum Method
 - ← Vogel Approximation Method
- Find T_k for this feasible plan and cross out all the unoccupied cells for which $t_{ij} \geq T_k$.
- Trace a closed path for the occupied cells corresponding to T_k . If no such closed path can be formed, the solution obtained is optimum otherwise, go to step 2.

Example 1

The following matrix gives data concerning the transportation times t_{ij}

Origin	Destination						Supply
	D1	D2	D3	D4	D5	D6	
O1	25	30	20	40	45	37	37
O2	30	25	20	30	40	20	22
O3	40	20	40	35	45	22	32
O4	25	24	50	27	30	25	14
Demand	15	20	15	25	20	10	

Solution.

We compute an initial basic feasible solution by north west corner rule which is shown in table 1.

Table 1

Origin	Destination						Supply
	D1	D2	D3	D4	D5	D6	
O1	25 ¹⁵	30 ²⁰	20 ²	40	45	37	37
O2	30	25	20 ¹³	30 ⁹	40	20	22
O3	40	20	40	35 ¹⁶	45 ¹⁶	22	32
O4	25	24	50	27	30 ⁴	25 ¹⁰	14
Demand	15	20	15	25	20	10	

Here, $t_{11} = 25$, $t_{12} = 30$, $t_{13} = 20$, $t_{23} = 20$, $t_{24} = 30$, $t_{34} = 35$, $t_{35} = 45$, $t_{45} = 30$, $t_{46} = 25$

Choose maximum from t_{ij} , i.e., $T_1 = 45$. Now, cross out all the unoccupied cells that are $\geq T_1$.

The unoccupied cell (O3D6) enters into the basis as shown in table 2.

Table 2

Origin	Destination						Supply
	D1	D2	D3	D4	D5	D6	
O1	25 (15)	30 (20)	20 (2)	40	45	37	37
O2	30	25	20 (13)	30 (9)	40	20	22
O3	40	20	40	35 (16)	45 (16)	+ 22	32
O4	25	24	50	27	+ 30 (4)	- 25 (10)	14
Demand	15	20	15	25	20	10	

Choose the smallest value with a negative position on the closed path, i.e., 10. Clearly only 10 units can be shifted to the entering cell. The next feasible plan is shown in the following table.

Table 3

Origin	Destination						Supply
	D1	D2	D3	D4	D5	D6	
O1	25 (15)	30 (20)	20 (2)	40	45	37	37
O2	30	25	20 (13)	30 (9)	40	20	22
O3	40	20	40	35 (16)	45 (6)	22 (10)	32
O4	25	24	50	27	30 (14)	25	14
Demand	15	20	15	25	20	10	

Here, $T_2 = \text{Max}(25, 30, 20, 20, 20, 35, 45, 22, 30) = 45$. Now, cross out all the unoccupied cells that are $\geq T_2$.

Table 4

Origin	Destination						Supply
	D1	D2	D3	D4	D5	D6	
O1	25 (15)	30 (20)	20 (2)	40	45	37	37
O2	30	25	20 (13)	- 30 (9) + 40		20	22
O3	40	20	40	+ 35 (16) - 45 (6)		22 (10)	32
O4	25	24	50	27	30 (14)	25	14
Demand	15	20	15	25	20	10	

By following the same procedure as explained above, we get the following revised matrix.

Table 6

Origin	Destination						Supply
	D1	D2	D3	D4	D5	D6	
O1	25 (15)	30 (20)	20 (2)	40	45	37	37
O2	30	25	20 (13)	30 (3)	40 (6)	20	22
O3	40	20	40	35 (22)	45	22 (10)	32
O4	25	24	50	27	30 (14)	25	14
Demand	15	20	15	25	20	10	

$T_3 = \text{Max}(25, 30, 20, 20, 30, 40, 35, 22, 30) = 40$. Now, cross out all the unoccupied cells that are $\geq T_3$.

Now we cannot form any other closed loop with T_3 .
Hence, the solution obtained at this stage is optimal.
Thus, all the shipments can be made within 40 units.