

44. The Southeastern Conference has nine basketball officials who must be assigned to three conference games, three to each game. The conference office wants to assign the officials so that the total distance they travel will be minimized. The distance (in miles) each official would travel to each game is given in the following table.

Official	Game		
	Athens	Columbia	Nashville
1	165	90	130
2	75	210	320
3	180	170	140
4	220	80	60
5	410	140	80
6	150	170	190
7	170	110	150
8	105	125	160
9	240	200	155

a) Determine the optimal assignment(s) to minimize the total distance traveled by the officials.

b) The officials 2 and 8 recently had a confrontation with one of the coaches in the game in Athens. They were forced to eject the coach after several technical fouls. The conference office decided that it would not be a good idea to have these two officials work the Athens game so soon after this confrontation, so they decided that officials 2 and 8 will not be assigned to the Athens game. How will this affect the optimal solution to this problem?

Find Solution using Least Cost method, also find optimal solution using stepping stone method

	D1	D2	D3	Supply
S1	165	90	130	1
S2	75	210	320	1
S3	180	170	140	1
S4	220	80	60	1
S5	410	140	80	1
S6	150	170	190	1
S7	170	110	150	1
S8	105	125	160	1
S9	240	200	155	1
Demand	3	3	3	

min

165X1A+75X2A+180X3A+220X4A+410X5A+150X6A+170X7A+105X8A+240X9A +
90X1B+210X2B+170X3B+80X4B+140X5B+170X6B+110X7B+125X8B+200X9B +
130X1C+320X2C+140X3C+60X4C+80X5C+190X6C+150X7C+160X8C+155X9C

s. t.

1S) X1A +X1B +X1C <= 1

2S) X2A +X2B +X2C <= 1

3S) X3A +X3B +X3C <= 1

4S) X4A +X4B +X4C <= 1

5S) X5A +X5B +X5C <= 1

6S) X6A +X6B +X6C <= 1

7S) X7A +X7B +X7C <= 1

8S) X8A +X8B +X8C <= 1

9S) X9A +X9B +X9C <= 1

1D) X1A+X2A+X3A+X4A+X5A+X6A+X7A+X8A+X9A=3

2D) X1B+X2B+X3B+X4B+X5B+X6B+X7B+X8B+X9B=3

3D) X1C+X2C+X3C+X4C+X5C+X6C+X7C+X8C+X9C=3

LP OPTIMUM FOUND AT STEP 12

OBJECTIVE FUNCTION VALUE

1) 985.0000

VARIABLE	VALUE	REDUCED COST
X1A	0	95
X2A	1	0
X3A	0	40
X4A	0	160
X5A	0	330
X6A	1	0
X7A	0	80
X8A	1	0
X9A	0	85
X1B	1	0
X2B	0	115
X3B	0	10
X4B	1	0
X5B	0	40
X6B	0	0
X7B	1	0
X8B	0	0
X9B	0	25
X1C	0	60
X2C	0	245
X3C	1	0
X4C	0	0
X5C	1	0
X6C	0	40
X7C	0	60
X8C	0	55
X9C	1	0

Solution:

TOTAL number of supply constraints : 9

TOTAL number of demand constraints : 3

Problem Table is

	D1	D2	D3	Supply
S1	165	90	130	1
S2	75	210	320	1
S3	180	170	140	1
S4	220	80	60	1
S5	410	140	80	1
S6	150	170	190	1
S7	170	110	150	1
S8	105	125	160	1
S9	240	200	155	1
Demand	3	3	3	

The smallest transportation cost is 60 in cell S4D3

The allocation to this cell is $\min(1,3) = 1$.

This exhausts the capacity of S4 and leaves $3 - 1 = 2$ units with D3

Table-1

	D1	D2	D3	Supply
S1	165	90	130	1
S2	75	210	320	1
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80	1
S6	150	170	190	1
S7	170	110	150	1
S8	105	125	160	1
S9	240	200	155	1
Demand	3	3	2	

The smallest transportation cost is 75 in cell S2D1

The allocation to this cell is $\min(1,3) = 1$.

This exhausts the capacity of S2 and leaves $3 - 1 = 2$ units with D1

Table-2

	D1	D2	D3	Supply
S1	165	90	130	1
S2	75(1)	210	320	0
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80	1
S6	150	170	190	1
S7	170	110	150	1
S8	105	125	160	1
S9	240	200	155	1
Demand	2	3	2	

The smallest transportation cost is 80 in cell S5D3

The allocation to this cell is $\min(1,2) = 1$.

This exhausts the capacity of S5 and leaves $2 - 1 = 1$ units with D3

Table-3

	D1	D2	D3	Supply
S1	165	90	130	1
S2	75(1)	210	320	0
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110	150	1
S8	105	125	160	1
S9	240	200	155	1
Demand	2	3	1	

The smallest transportation cost is 90 in cell S1D2

The allocation to this cell is $\min(1,3) = 1$.

This exhausts the capacity of S1 and leaves $3 - 1 = 2$ units with D2

Table-4

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	75(1)	210	320	0
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110	150	1
S8	105	125	160	1
S9	240	200	155	1
Demand	2	2	1	

The smallest transportation cost is 105 in cell S8D1

The allocation to this cell is $\min(1,2) = 1$.

This exhausts the capacity of S8 and leaves $2 - 1 = 1$ units with D1

Table-5

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	75(1)	210	320	0
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110	150	1
S8	105(1)	125	160	0
S9	240	200	155	1
Demand	1	2	1	

The smallest transportation cost is 110 in cell S7D2

The allocation to this cell is $\min(1,2) = 1$.

This exhausts the capacity of S7 and leaves $2 - 1 = 1$ units with D2

Table-6

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	75(1)	210	320	0
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110(1)	150	0
S8	105(1)	125	160	0
S9	240	200	155	1
Demand	1	1	1	

The smallest transportation cost is 140 in cell S3D3

The allocation to this cell is $\min(1,1) = 1$.

Table-7

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	75(1)	210	320	0
S3	180	170	140(1)	0
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110(1)	150	0
S8	105(1)	125	160	0
S9	240	200	155	1
Demand	1	1	0	

The smallest transportation cost is 150 in cell S6D1

The allocation to this cell is $\min(1,1) = 1$.

Table-8

The smallest transportation cost is 200 in cell S9D2
 The allocation to this cell is $\min(1,1) = 1$.

Table-9

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	75(1)	210	320	0
S3	180	170	140(1)	0
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150(1)	170	190	0
S7	170	110(1)	150	0
S8	105(1)	125	160	0
S9	240	200(1)	155	0
Demand	0	0	0	

Initial feasible solution is

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	75 (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	105 (1)	125	160	1
S9	240	200 (1)	155	1
Demand	3	3	3	

The minimum total transportation cost

$$=90 \times 1 + 75 \times 1 + 140 \times 1 + 60 \times 1 + 80 \times 1 + 150 \times 1 + 110 \times 1 + 105 \times 1 + 200 \times 1 = 1010$$

Here, the number of allocated cells = 9, which is two less than to $m + n - 1 = 9 + 3 - 1 = 11$
 \therefore This solution is degenerate

To resolve degeneracy, we make use of an artificial quantity(d).

The quantity d is assigned to that unoccupied cell, which has the minimum transportation cost.

The quantity d is assigned to S4D2, which has the minimum transportation cost = 80.

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	75 (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (d)	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	105 (1)	125	160	1
S9	240	200 (1)	155	1
Demand	3	3	3	

Optimality test using stepping stone method...

Allocation Table is

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	75 (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (d)	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	105 (1)	125	160	1
S9	240	200 (1)	155	1
Demand	3	3	3	

Here, the number of allocated cells = 10, which is one less than to $m + n - 1 = 9 + 3 - 1 = 11$

∴ This solution is degenerate

The quantity d is assigned to S8D2, which has the minimum transportation cost = 125.

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	75 (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (d)	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	105 (1)	125 (d)	160	1
S9	240	200 (1)	155	1
Demand	3	3	3	

Iteration-1 of optimality test

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S1D1	S1D1→S1D2→S8D2→S8D1	$165 - 90 + 125 - 105=95$
S1D3	S1D3→S1D2→S4D2→S4D3	$130 - 90 + 80 - 60=60$
S2D2	S2D2→S2D1→S8D1→S8D2	$210 - 75 + 105 - 125=115$
S2D3	S2D3→S2D1→S8D1→S8D2→S4D2→S4D3	$320 - 75 + 105 - 125 + 80 - 60=245$
S3D1	S3D1→S3D3→S4D3→S4D2→S8D2→S8D1	$180 - 140 + 60 - 80 + 125 - 105=40$
S3D2	S3D2→S3D3→S4D3→S4D2	$170 - 140 + 60 - 80=10$
S4D1	S4D1→S4D2→S8D2→S8D1	$220 - 80 + 125 - 105=160$
S5D1	S5D1→S5D3→S4D3→S4D2→S8D2→S8D1	$410 - 80 + 60 - 80 + 125 - 105=330$
S5D2	S5D2→S5D3→S4D3→S4D2	$140 - 80 + 60 - 80=40$
S6D2	S6D2→S6D1→S8D1→S8D2	$170 - 150 + 105 - 125=0$
S6D3	S6D3→S6D1→S8D1→S8D2→S4D2→S4D3	$190 - 150 + 105 - 125 + 80 - 60=40$
S7D1	S7D1→S7D2→S8D2→S8D1	$170 - 110 + 125 - 105=80$
S7D3	S7D3→S7D2→S4D2→S4D3	$150 - 110 + 80 - 60=60$
S8D3	S8D3→S8D2→S4D2→S4D3	$160 - 125 + 80 - 60=55$
S9D1	S9D1→S9D2→S8D2→S8D1	$240 - 200 + 125 - 105=60$
S9D3	S9D3→S9D2→S4D2→S4D3	$155 - 200 + 80 - 60=-25$

2. Select the unoccupied cell having the highest negative net cost change i.e. cell S9D3=-25. and draw a closed path from S9D3.

Closed path is S9D3→S9D2→S4D2→S4D3

Closed path and plus/minus allocation for current unoccupied cell S9D3

	D1	D2	D3	Supply
S1	165 [95]	90 (1)	130 [60]	1
S2	75 (1)	210 [115]	320 [245]	1
S3	180 [40]	170 [10]	140 (1)	1
S4	220 [160]	80 (d) (+)	60 (1) (-)	1
S5	410 [330]	140 [40]	80 (1)	1
S6	150 (1)	170 [0]	190 [40]	1
S7	170 [80]	110 (1)	150 [60]	1
S8	105 (1)	125 (d)	160 [55]	1
S9	240 [60]	200 (1) (-)	155 [-25] (+)	1
Demand	3	3	3	

3. Minimum allocated value among all negative position (-) on closed path = 1
Subtract 1 from all (-) and Add it to all (+)

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	75 (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (1)	60	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	105 (1)	125 (d)	160	1
S9	240	200	155 (1)	1
Demand	3	3	3	

4. Repeat the step 1 to 3, until an optimal solution is obtained.

Here, the number of allocated cells = 10, which is one less than to $m + n - 1 = 9 + 3 - 1 = 11$
∴ This solution is degenerate

The quantity d is assigned to S4D3, which has the minimum transportation cost = 60.

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	75 (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (1)	60 (d)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	105 (1)	125 (d)	160	1
S9	240	200	155 (1)	1
Demand	3	3	3	

Iteration-2 of optimality test

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S1D1	S1D1→S1D2→S8D2→S8D1	165 - 90 + 125 - 105=95
S1D3	S1D3→S1D2→S4D2→S4D3	130 - 90 + 80 - 60=60
S2D2	S2D2→S2D1→S8D1→S8D2	210 - 75 + 105 - 125=115
S2D3	S2D3→S2D1→S8D1→S8D2→S4D2→S4D3	320 - 75 + 105 - 125 + 80 - 60=245
S3D1	S3D1→S3D3→S4D3→S4D2→S8D2→S8D1	180 - 140 + 60 - 80 + 125 - 105=40
S3D2	S3D2→S3D3→S4D3→S4D2	170 - 140 + 60 - 80=10
S4D1	S4D1→S4D2→S8D2→S8D1	220 - 80 + 125 - 105=160
S5D1	S5D1→S5D3→S4D3→S4D2→S8D2→S8D1	410 - 80 + 60 - 80 + 125 - 105=330
S5D2	S5D2→S5D3→S4D3→S4D2	140 - 80 + 60 - 80=40
S6D2	S6D2→S6D1→S8D1→S8D2	170 - 150 + 105 - 125=0
S6D3	S6D3→S6D1→S8D1→S8D2→S4D2→S4D3	190 - 150 + 105 - 125 + 80 - 60=40
S7D1	S7D1→S7D2→S8D2→S8D1	170 - 110 + 125 - 105=80
S7D3	S7D3→S7D2→S4D2→S4D3	150 - 110 + 80 - 60=60
S8D3	S8D3→S8D2→S4D2→S4D3	160 - 125 + 80 - 60=55
S9D1	S9D1→S9D3→S4D3→S4D2→S8D2→S8D1	240 - 155 + 60 - 80 + 125 - 105=85
S9D2	S9D2→S9D3→S4D3→S4D2	200 - 155 + 60 - 80=25

Since all net cost change ≥ 0

So final optimal solution is arrived.

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	75 (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (1)	60 (d)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	105 (1)	125 (d)	160	1
S9	240	200	155 (1)	1
Demand	3	3	3	

The minimum total transportation cost
 $=90 \times 1 + 75 \times 1 + 140 \times 1 + 80 \times 1 + 80 \times 1 + 150 \times 1 + 110 \times 1 + 105 \times 1 + 155 \times 1 = 985$

Notice alternate solution is available with unoccupied cell S6D2=0, but with the same optimal value.

Find Solution of Assignment problem using Hungarian method

Work \ Job	A	C	N	A	C	N	A	C	N
1	165	90	130	165	90	130	165	90	130
2	75	210	320	75	210	320	75	210	320
3	180	170	140	180	170	140	180	170	140
4	220	80	60	220	80	60	220	80	60
5	410	140	80	410	140	80	410	140	80
6	150	170	190	150	170	190	150	170	190
7	170	110	150	170	110	150	170	110	150
8	105	125	160	105	125	160	105	125	160
9	240	200	155	240	200	155	240	200	155

Optimal solution is

Work	Job	Cost
1	C	90
2	A	75
3	N	140
4	C	80
5	N	80
6	A	150
7	C	110
8	A	105
9	N	155
	Total	985

b) The officials 2 and 8 recently had a confrontation with one of the coaches in the game in Athens. They were forced to eject the coach after several technical fouls. The conference office decided that it would not be a good idea to have these two officials work the Athens game so soon after this confrontation, so they decided that officials 2 and 8 will not be assigned to the Athens game. How will this affect the optimal solution to this problem?

min $165X_{1A}+180X_{3A}+220X_{4A}+410X_{5A}+150X_{6A}+170X_{7A}+240X_{9A} +$
 $90X_{1B}+210X_{2B}+170X_{3B}+80X_{4B}+140X_{5B}+170X_{6B}+110X_{7B}+125X_{8B}+200X_{9B} +$
 $130X_{1C}+320X_{2C}+140X_{3C}+60X_{4C}+80X_{5C}+190X_{6C}+150X_{7C}+160X_{8C}+155X_{9C}$
s. t.

- 1S) $X_{1A} + X_{1B} + X_{1C} \leq 1$
- 2S) $X_{2B} + X_{2C} \leq 1$
- 3S) $X_{3A} + X_{3B} + X_{3C} \leq 1$
- 4S) $X_{4A} + X_{4B} + X_{4C} \leq 1$
- 5S) $X_{5A} + X_{5B} + X_{5C} \leq 1$
- 6S) $X_{6A} + X_{6B} + X_{6C} \leq 1$
- 7S) $X_{7A} + X_{7B} + X_{7C} \leq 1$
- 8S) $X_{8B} + X_{8C} \leq 1$
- 9S) $X_{9A} + X_{9B} + X_{9C} \leq 1$
- 1D) $X_{1A}+X_{3A}+X_{4A}+X_{5A}+X_{6A}+X_{7A}+X_{9A}=3$
- 2D) $X_{1B}+X_{2B}+X_{3B}+X_{4B}+X_{5B}+X_{6B}+X_{7B}+X_{8B}+X_{9B}=3$
- 3D) $X_{1C}+X_{2C}+X_{3C}+X_{4C}+X_{5C}+X_{6C}+X_{7C}+X_{8C}+X_{9C}=3$

LP OPTIMUM FOUND AT STEP 14
OBJECTIVE FUNCTION VALUE
1) 1220

VARIABLE	VALUE	REDUCED COST
X1A	0	15
X3A	1	0
X4A	0	80
X5A	0	250
X6A	1	0
X7A	1	0
X9A	0	5
X1B	1	0
X2B	1	0
X3B	0	50
X4B	0	0
X5B	0	40
X6B	0	80
X7B	0	0
X8B	1	0
X9B	0	25
X1C	0	60
X2C	0	130
X3C	0	40
X4C	1	0
X5C	1	0
X6C	0	120
X7C	0	60
X8C	0	55
X9C	1	0

Step-2: Find out the each column minimum element and subtract it from that column.

	A	C	N	A	C	N	A	C	N	
1	75	0	40	75	0	40	75	0	40	
2	M	0	110	M	0	110	M	0	110	
3	40	30	0	40	30	0	40	30	0	
4	160	20	0	160	20	0	160	20	0	
5	330	60	0	330	60	0	330	60	0	
6	0	20	40	0	20	40	0	20	40	
7	60	0	40	60	0	40	60	0	40	
8	M	0	35	M	0	35	M	0	35	
9	85	45	0	85	45	0	85	45	0	
	(-0)	(-0)	(-0)	(-0)	(-0)	(-0)	(-0)	(-0)	(-0)	

Step-3: Cover all zeros with a minimum number of lines

Determine the minimum number of lines, required to cover all zeros in the matrix.

	A	C	N	A	C	N	A	C	N	
1	75	0	40	75	0	40	75	0	40	
2	M	0	110	M	0	110	M	0	110	
3	40	30	0	40	30	0	40	30	0	
4	160	20	0	160	20	0	160	20	0	
5	330	60	0	330	60	0	330	60	0	
6	0	20	40	0	20	40	0	20	40	√
7	60	0	40	60	0	40	60	0	40	
8	M	0	35	M	0	35	M	0	35	
9	85	45	0	85	45	0	85	45	0	
		√	√		√	√		√	√	

Step-4: Create additional zeros, Develop the new revised table by selecting the smallest element, among the cells not covered by any line (say $k = 40$)

Subtract $k = 40$ from every element in the cell not covered by a line.

Add $k = 40$ to every element in the intersection cell of two lines.

	A	C	N	A	C	N	A	C	N	
1	35	0	40	35	0	40	35	0	40	
2	M	0	110	M	0	110	M	0	110	
3	0	30	0	0	30	0	0	30	0	
4	120	20	0	120	20	0	120	20	0	
5	290	60	0	290	60	0	290	60	0	
6	0	60	80	0	60	80	0	60	80	
7	20	0	40	20	0	40	20	0	40	
8	M	0	35	M	0	35	M	0	35	
9	45	45	0	45	45	0	45	45	0	

Step-3: Cover all zeros with a minimum number of lines

Determine the minimum number of lines, required to cover all zeros in the matrix.

	A	C	N	A	C	N	A	C	N	
1	35	0	40	35	0	40	35	0	40	
2	M	0	110	M	0	110	M	0	110	
3	0	30	0	0	30	0	0	30	0	√
4	120	20	0	120	20	0	120	20	0	
5	290	60	0	290	60	0	290	60	0	
6	0	60	80	0	60	80	0	60	80	√
7	20	0	40	20	0	40	20	0	40	
8	M	0	35	M	0	35	M	0	35	
9	45	45	0	45	45	0	45	45	0	
		√	√		√	√		√	√	

The Optimal assignments are

	A	C	N	A	C	N	A	C	N	
1	15	0	40	15	0	40	15	[0]	40	
2	M	0	110	M	[0]	110	M	0	110	
3	0	50	20	0	50	20	[0]	50	20	
4	100	20	0	100	20	[0]	100	20	0	
5	270	60	0	270	60	0	270	60	[0]	
6	0	80	100	[0]	80	100	0	80	100	
7	[0]	0	40	0	0	40	0	0	40	
8	M	[0]	35	M	0	35	M	0	35	
9	25	45	[0]	25	45	0	25	45	0	

Optimal solution is

Work	Job	Cost
1	C	90
2	C	210
3	A	180
4	N	60
5	N	80
6	A	150
7	A	170
8	C	125
9	N	155
	Total	1220

Find Solution using Least Cost method, also find optimal solution using stepping stone method

	D1	D2	D3	Supply
S1	165	90	130	1
S2	M	210	320	1
S3	180	170	140	1
S4	220	80	60	1
S5	410	140	80	1
S6	150	170	190	1
S7	170	110	150	1
S8	M	125	160	1
S9	240	200	155	1
Demand	3	3	3	

Solution:

TOTAL number of supply constraints : 9

TOTAL number of demand constraints : 3

Problem Table is

	D1	D2	D3	Supply
S1	165	90	130	1
S2	M	210	320	1
S3	180	170	140	1
S4	220	80	60	1
S5	410	140	80	1
S6	150	170	190	1
S7	170	110	150	1
S8	M	125	160	1
S9	240	200	155	1
Demand	3	3	3	

The smallest transportation cost is 60 in cell S4D3

The allocation to this cell is $\min(1,3) = 1$.

This exhausts the capacity of S4 and leaves $3 - 1 = 2$ units with D3

Table-1

	D1	D2	D3	Supply
S1	165	90	130	1
S2	M	210	320	1
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80	1
S6	150	170	190	1
S7	170	110	150	1
S8	M	125	160	1
S9	240	200	155	1
Demand	3	3	2	

The smallest transportation cost is 80 in cell S5D3

The allocation to this cell is $\min(1,2) = 1$.

This exhausts the capacity of S5 and leaves $2 - 1 = 1$ units with D3

Table-2

	D1	D2	D3	Supply
S1	165	90	130	1
S2	M	210	320	1
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110	150	1
S8	M	125	160	1
S9	240	200	155	1
Demand	3	3	1	

The smallest transportation cost is 90 in cell S1D2

The allocation to this cell is $\min(1,3) = 1$.

This exhausts the capacity of S1 and leaves $3 - 1 = 2$ units with D2

Table-3

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	M	210	320	1
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110	150	1
S8	M	125	160	1
S9	240	200	155	1
Demand	3	2	1	

The smallest transportation cost is 110 in cell S7D2

The allocation to this cell is $\min(1,2) = 1$.

This exhausts the capacity of S7 and leaves $2 - 1 = 1$ units with D2

Table-4

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	M	210	320	1
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110(1)	150	0
S8	M	125	160	1
S9	240	200	155	1
Demand	3	1	1	

The smallest transportation cost is 125 in cell S8D2

The allocation to this cell is $\min(1,1) = 1$.

Table-5

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	M	210	320	1
S3	180	170	140	1
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110(1)	150	0
S8	M	125(1)	160	0
S9	240	200	155	1
Demand	3	0	1	

The smallest transportation cost is 140 in cell S3D3

The allocation to this cell is $\min(1,1) = 1$.

Table-6

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	M	210	320	1
S3	180	170	140(1)	0
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150	170	190	1
S7	170	110(1)	150	0
S8	M	125(1)	160	0
S9	240	200	155	1
Demand	3	0	0	

The smallest transportation cost is 150 in cell S6D1

The allocation to this cell is $\min(1,3) = 1$.

This exhausts the capacity of S6 and leaves $3 - 1 = 2$ units with D1

Table-7

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	M	210	320	1
S3	180	170	140(1)	0
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150(1)	170	190	0
S7	170	110(1)	150	0
S8	M	125(1)	160	0
S9	240	200	155	1
Demand	2	0	0	

The smallest transportation cost is 240 in cell S9D1

The allocation to this cell is $\min(1,2) = 1$.

This exhausts the capacity of S9 and leaves $2 - 1=1$ units with D1

Table-8

	D1	D2	D3	Supply
S1	165	90(1)	130	0
S2	M	210	320	1
S3	180	170	140(1)	0
S4	220	80	60(1)	0
S5	410	140	80(1)	0
S6	150(1)	170	190	0
S7	170	110(1)	150	0
S8	M	125(1)	160	0
S9	240(1)	200	155	0
Demand	1	0	0	

The smallest transportation cost is **M** in cell S2D1

The allocation to this cell is $\min(1,1) = 1$.

Table-9

	D1	D2	D3	Supply
S1	165	90 (1)	130	0
S2	M(1)	210	320	0
S3	180	170	140 (1)	0
S4	220	80	60 (1)	0
S5	410	140	80 (1)	0
S6	150 (1)	170	190	0
S7	170	110 (1)	150	0
S8	M	125 (1)	160	0
S9	240 (1)	200	155	0
Demand	0	0	0	

Initial feasible solution is

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	M (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	M	125 (1)	160	1
S9	240 (1)	200	155	1
Demand	3	3	3	

The minimum total transportation cost

$$=90 \times 1 + M \times 1 + 140 \times 1 + 60 \times 1 + 80 \times 1 + 150 \times 1 + 110 \times 1 + 125 \times 1 + 240 \times 1 = M$$

Here, the number of allocated cells = 9, which is two less than to $m + n - 1 = 9 + 3 - 1 = 11$

\therefore This solution is degenerate

To resolve degeneracy, we make use of an artificial quantity(d).

The quantity d is assigned to that unoccupied cell, which has the minimum transportation cost.

The quantity d is assigned to S4D2, which has the minimum transportation cost = 80.

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	M (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (d)	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	M	125 (1)	160	1
S9	240 (1)	200	155	1
Demand	3	3	3	

Optimality test using stepping stone method...

Allocation Table is

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	M (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (d)	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	M	125 (1)	160	1
S9	240 (1)	200	155	1
Demand	3	3	3	

Here, the number of allocated cells = 10, which is one less than to $m + n - 1 = 9 + 3 - 1 = 11$

∴ This solution is degenerate

The quantity d is assigned to S9D3, which has the minimum transportation cost = 155.

	D1	D2	D3	Supply
S1	165	90 (1)	130	1
S2	M (1)	210	320	1
S3	180	170	140 (1)	1
S4	220	80 (d)	60 (1)	1
S5	410	140	80 (1)	1
S6	150 (1)	170	190	1
S7	170	110 (1)	150	1
S8	M	125 (1)	160	1
S9	240 (1)	200	155 (d)	1
Demand	3	3	3	

Iteration-1 of optimality test

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S1D1	S1D1 → S1D2 → S4D2 → S4D3 → S9D3 → S9D1	165 - 90 + 80 - 60 + 155 - 240=10
S1D3	S1D3 → S1D2 → S4D2 → S4D3	130 - 90 + 80 - 60=60
S2D2	S2D2 → S2D1 → S9D1 → S9D3 → S4D3 → S4D2	210 - M + 240 - 155 + 60 - 80=-M
S2D3	S2D3 → S2D1 → S9D1 → S9D3	320 - M + 240 - 155=-M
S3D1	S3D1 → S3D3 → S9D3 → S9D1	180 - 140 + 155 - 240=-45
S3D2	S3D2 → S3D3 → S4D3 → S4D2	170 - 140 + 60 - 80=10
S4D1	S4D1 → S4D3 → S9D3 → S9D1	220 - 60 + 155 - 240=75
S5D1	S5D1 → S5D3 → S9D3 → S9D1	410 - 80 + 155 - 240=245
S5D2	S5D2 → S5D3 → S4D3 → S4D2	140 - 80 + 60 - 80=40
S6D2	S6D2 → S6D1 → S9D1 → S9D3 → S4D3 → S4D2	170 - 150 + 240 - 155 + 60 - 80=85
S6D3	S6D3 → S6D1 → S9D1 → S9D3	190 - 150 + 240 - 155=125
S7D1	S7D1 → S7D2 → S4D2 → S4D3 → S9D3 → S9D1	170 - 110 + 80 - 60 + 155 - 240=-5
S7D3	S7D3 → S7D2 → S4D2 → S4D3	150 - 110 + 80 - 60=60
S8D1	S8D1 → S8D2 → S4D2 → S4D3 → S9D3 → S9D1	M - 125 + 80 - 60 + 155 - 240=M
S8D3	S8D3 → S8D2 → S4D2 → S4D3	160 - 125 + 80 - 60=55
S9D2	S9D2 → S9D3 → S4D3 → S4D2	200 - 155 + 60 - 80=25

2. Select the unoccupied cell having the highest negative net cost change i.e. cell S2D2=-M.

and draw a closed path from S2D2.

Closed path is S2D2 → S2D1 → S9D1 → S9D3 → S4D3 → S4D2

Closed path and plus/minus allocation for current unoccupied cell S2D2

	D1	D2	D3	Supply
S1	165 [10]	90 (1)	130 [60]	1
S2	M (1) (-)	210 [-M] (+)	320 [-M]	1
S3	180 [-45]	170 [10]	140 (1)	1
S4	220 [75]	80 (d) (-)	60 (1) (+)	1
S5	410 [245]	140 [40]	80 (1)	1
S6	150 (1)	170 [85]	190 [125]	1
S7	170 [-5]	110 (1)	150 [60]	1
S8	M [M]	125 (1)	160 [55]	1
S9	240 (1) (+)	200 [25]	155 (d) (-)	1
Demand	3	3	3	

dummy cell is selected, so stop the procedure for current selected dummy cell.