ΕΧΝ210: ΜΑΘΗΜΑΤΙΚΕΣ ΜΕΘΟΔΟΙ ΣΤΑ ΟΙΚΟΝΟΜΙΚΑ ΚΑΙ ΔΙΟΙΚΗΣΗ ΙΙ

ΓΡΑΦΙΚΗ ΜΕΘΟΔΟΣ ΕΠΙΛΥΣΗΣ ΓΡΑΜΜΙΚΩΝ ΠΡΟΓΡΑΜΜΑΤΩΝ

Σημειώσεις από to ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΥΠΡΟΥ (ΔΙΟΙΚΗΤΙΚΗ ΕΠΙΣΤΗΜΗ)

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Problem Description

RMC, Inc. is a firm that produces chemical-based products. In a particular process three raw materials are used to produce two products. The material requirements per ton are shown below:

Product	Material 1	Material 2	Material 3
Fuel additive	2/5	0	3/5
Solvent base	1/2	1/5	3/10

Example: 1/2 ton material 1 is used in each ton of solvent base.

For the current production period RMC has available the following quantities of each raw material. Because of spoilage, any materials not used for current production must be discarded.

Material	Number of Tons Available for Production
Material 1	20
Material 2	5
Material 3	21

If the contribution to profit is $40 for each ton of fuel additive and $30 for each ton of solvent base, how many tons of each product should be produced in order to maximize the total contribution to profit?

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The Objective Function:

Let

x_1 = the number of tons of fuel additive that RMC produces

x_2 = the number of tons of solvent base that RMC produces

RMC's total contribution to profit will come from two sources:

1. The contribution to profit from producing x_1 tons of fuel additive
2. The contribution to profit from producing x_2 tons of solvent base

Therefore,

Total contribution to profit = 40 x_1 + 30 x_2

We say that RMC's objective is to maximize the value of its objective function. Using max as an abbreviation for maximize, the objective function is written as follows:

$$\max \; 40x_1 + 30x_2$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The Constraints:

Since there are limited amounts of the three raw materials available, we have three constraints that limit the amount of fuel additive and solvent base that RMC can produce.

Let us consider the constraint involving material 1.

Total tons of material 1 required is:

$$\frac{2}{5}x_1 + \frac{1}{2}x_2$$

Since 20 tons of material 1 are available, the production combination we select must satisfy the requirement:

$$\frac{2}{5}x_1 + \frac{1}{2}x_2 \leq 20$$

This expression is referred to as the material 1 constraint.

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The constraint for material 2 is:

$$\frac{1}{5}x_2 \le 5$$

The constraint for material 3 is:

$$\frac{3}{5}x_1 + \frac{3}{10}x_2 \le 21$$

Finally, since the decision variables cannot be negative, we add the nonnegative constraints:

$$x_1 \ge 0 \text{ and } x_2 \ge 0$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The mathematical statement or formulation of the RMC problem is now complete.

The mathematical model we developed can be written as follows:

$$\max \ 40x_1 + 30x_2$$

$$\text{subject to (s.t.)}$$

$$\frac{2}{5}x_1 + \frac{1}{2}x_2 \le 20 \quad \text{Material 1}$$

$$\frac{1}{5}x_2 \le 5 \quad \text{Material 2}$$

$$\frac{3}{5}x_1 + \frac{3}{10}x_2 \le 21 \quad \text{Material 3}$$

$$x_1 \ge 0, x_2 \ge 0$$

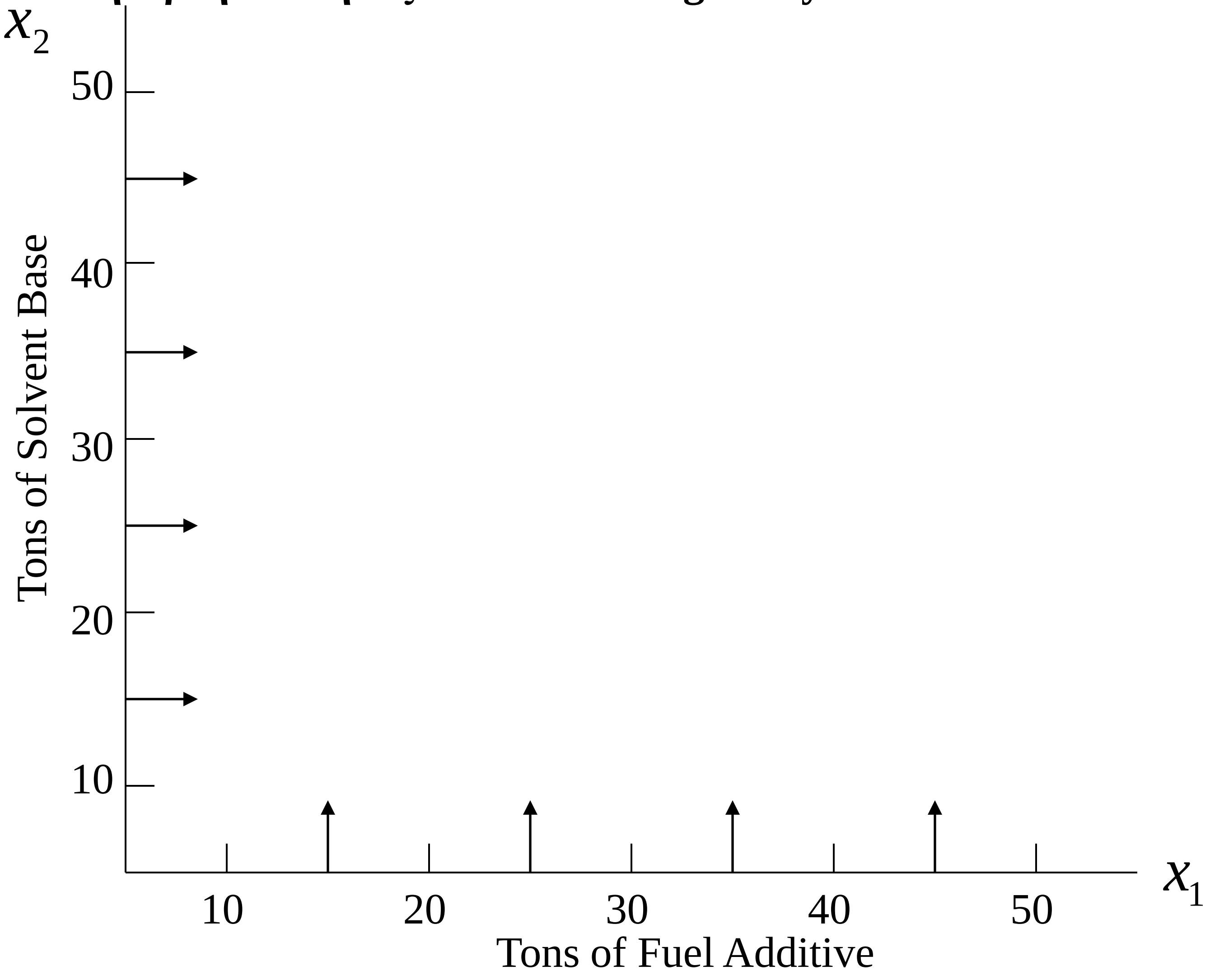
where

x_1 = the number of tons of fuel additive that RMC produces

x_2 = the number of tons of solvent base that RMC produces

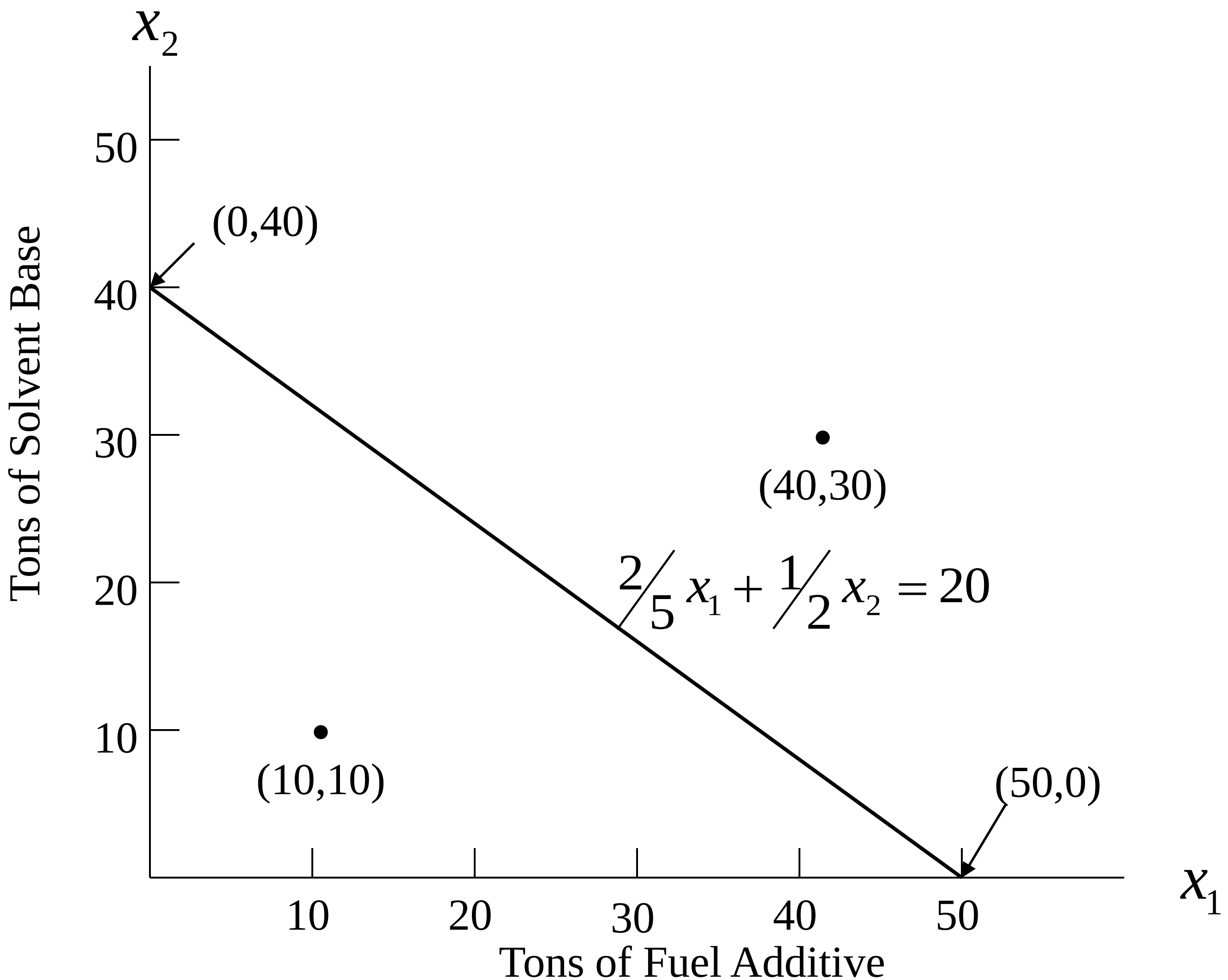
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Περιορισμοί Μη Αρνητικότητας -- The Non-negativity Constraints



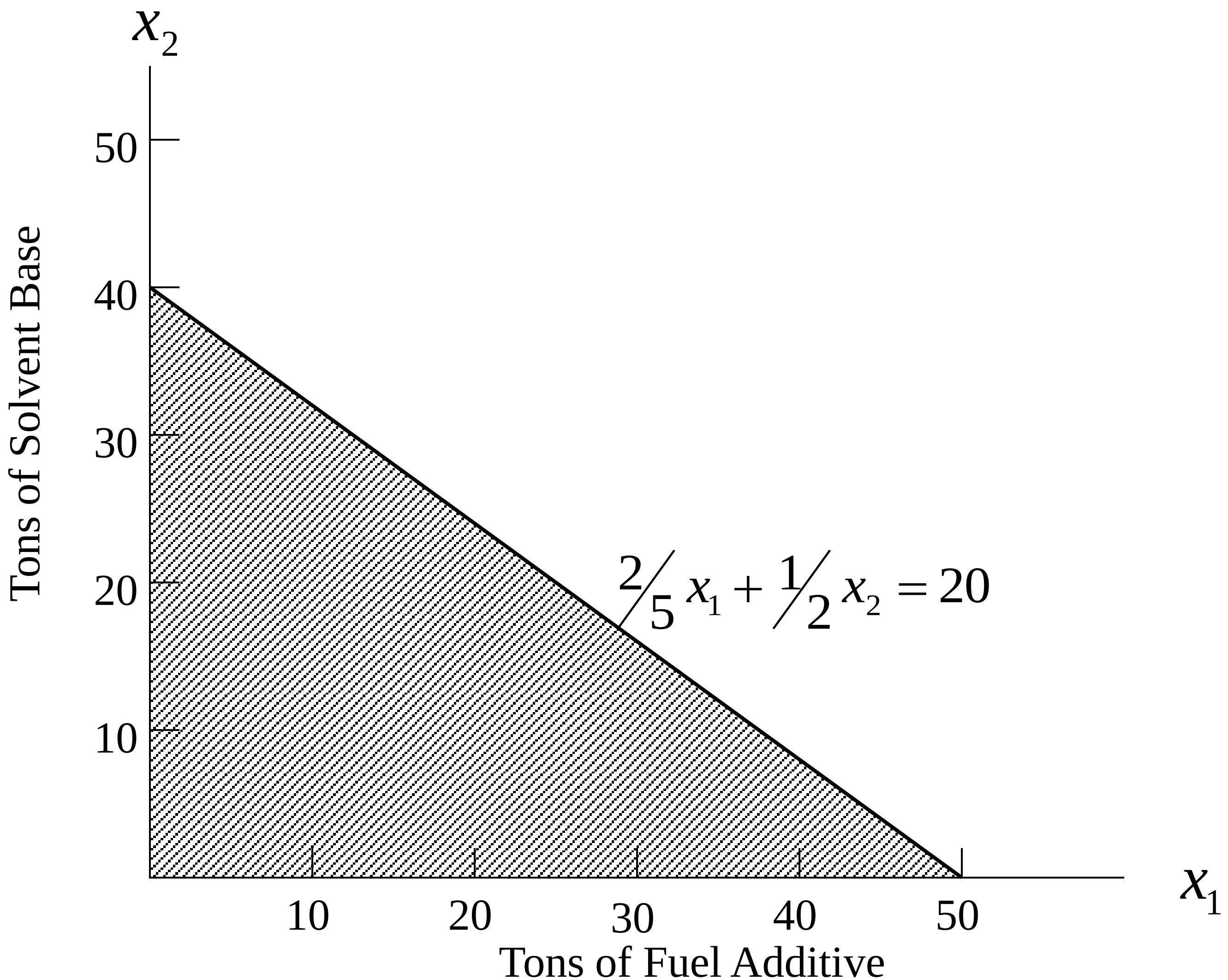
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The Material 1 Constraint Line

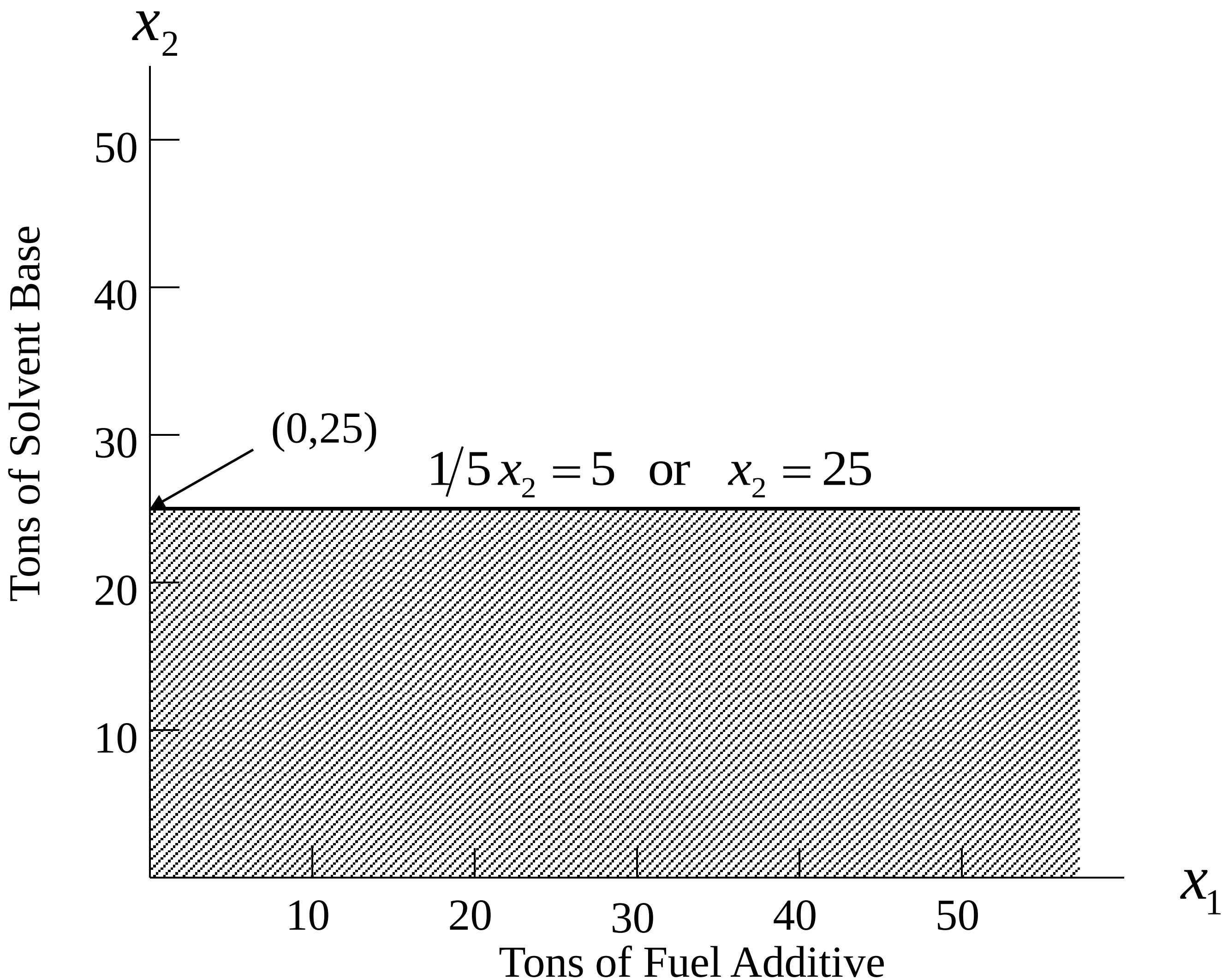


ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Feasible Region for the Material 1 Constraint

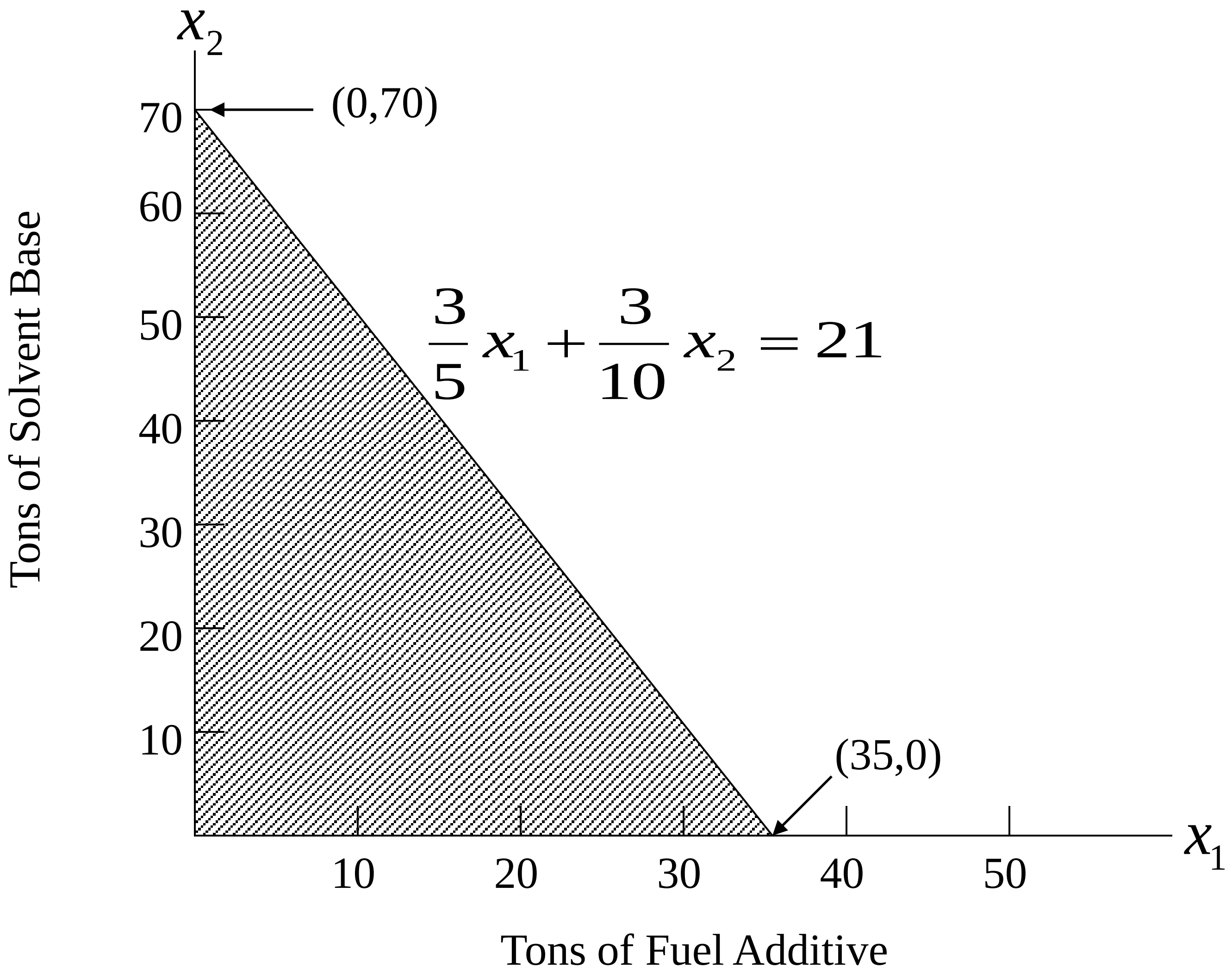


Feasible Region for the Material 2 Constraint



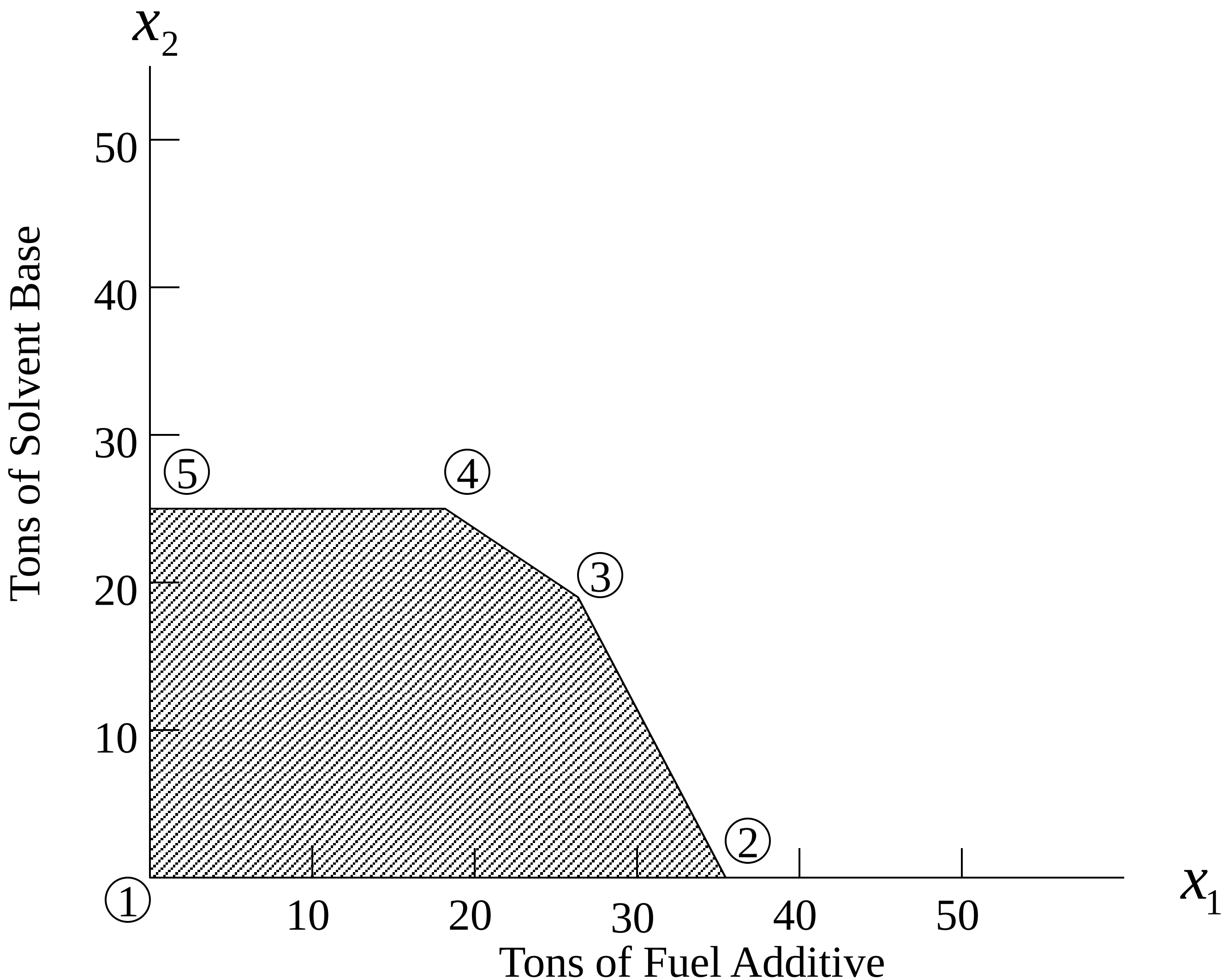
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Feasible Region for the Material 3 Constraint

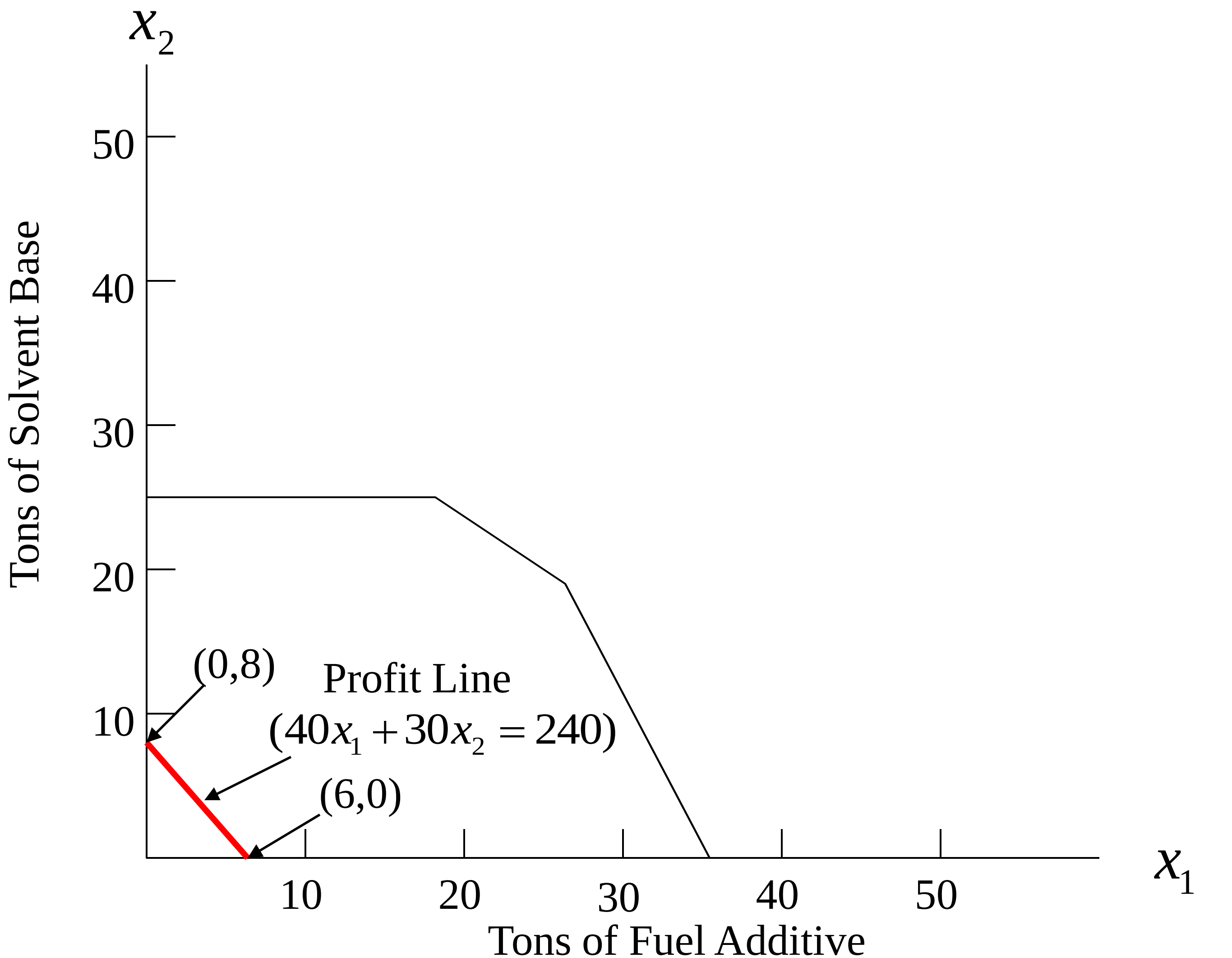


ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

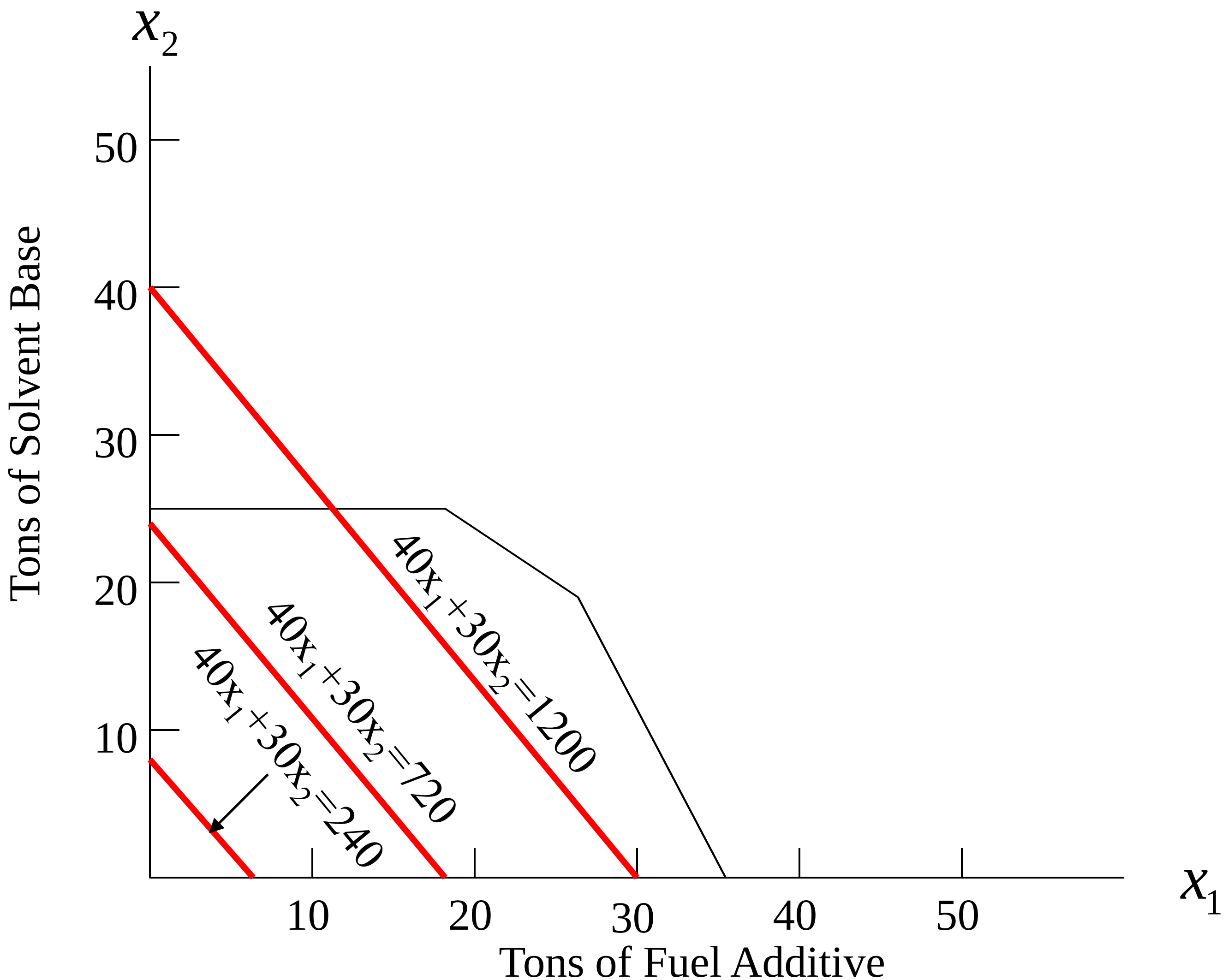
Feasible Region (Εφικτή περιοχή) for the RMC Problem



$240 Profit for the RMC Problem

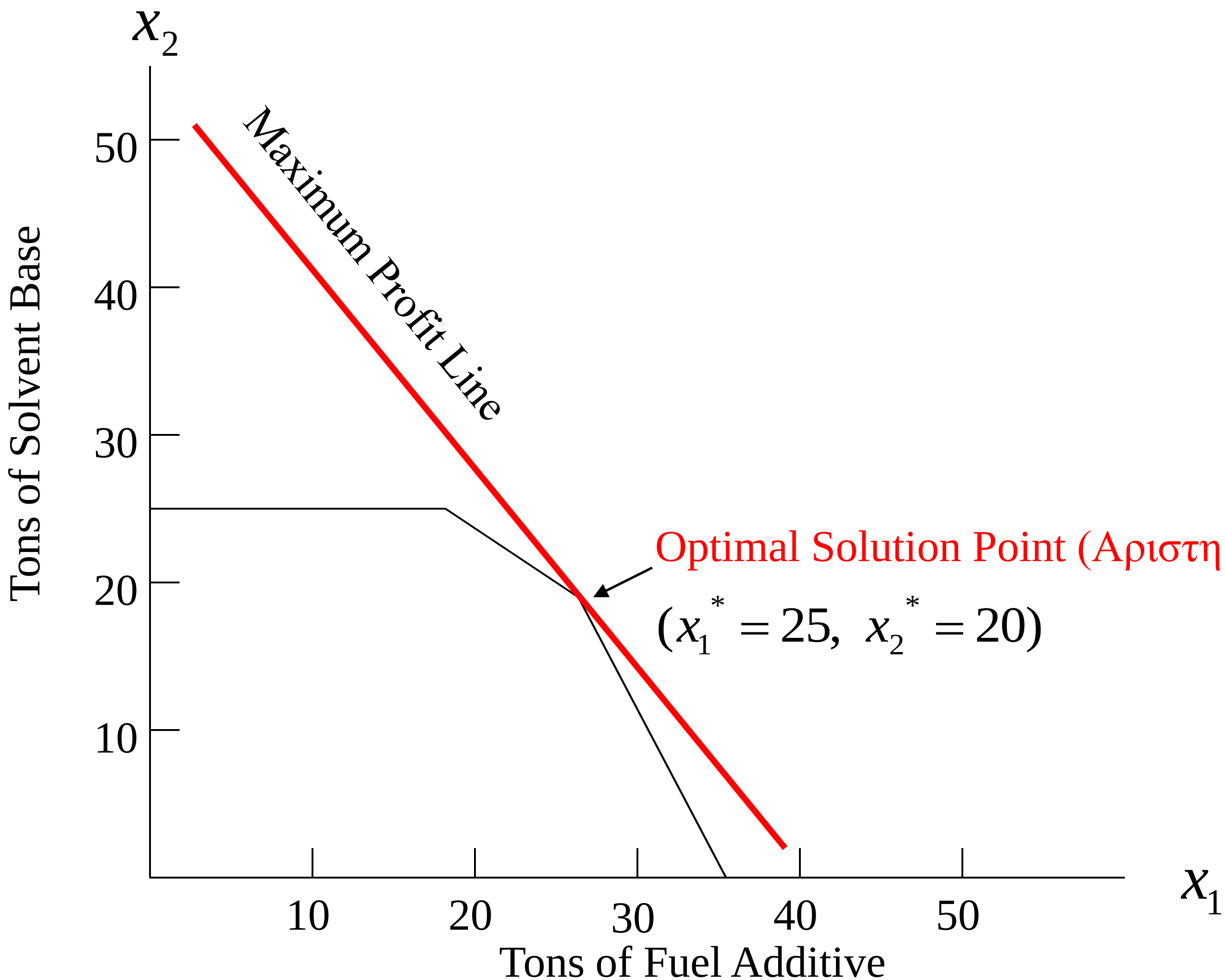


Selected Profit Lines for the RMC Problem



ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The Optimal Solution to the RMC Problem



ΛΥΣΗ ΓΡΑΜΜΙΚΩΝ ΠΡΟΓΡΑΜΜΑΤΩΝ
SOLUTION OF LPs

- **Η βέλτιστη (άριστη) λύση γραμμικού προγράμματος βρίσκεται σε ακραίο σημείο της εφικτής περιοχής του προβλήματος.**
  – An optimal solution to a linear programming problem can be found at an extreme point of the feasible region for the problem.

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Determining the optimal solution algebraically

The optimal solution is at the intersection of the material 1 constraint line and the material 3 constraint line. Thus, the values of the decision variables must satisfy the following equations simultaneously:

$$\frac{2}{5}x_1 + \frac{1}{2}x_2 = 20$$

$$\frac{3}{5}x_1 + \frac{3}{10}x_2 = 21$$

Solving for x_1 in the first equation we obtain:

$$\frac{2}{5}x_1 = 20 - \frac{1}{2}x_2 \quad \text{or} \quad x_1 = 50 - \frac{5}{4}x_2$$

Substituting this expression for x_1 into the second equation provides the following:

$$\frac{3}{5}(50 - \frac{5}{4}x_2) + \frac{3}{10}x_2 = 21 \Rightarrow \frac{9}{20}x_2 = 9$$

which we can solve to obtain $x_2 = 20$ Thus, using $x_2 = 20$ in the first equation and solving for x_1 we obtain:

$$x_1 = 50 - \frac{5}{4}(20) = 25$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Summary of optimal solution

Materials	**Tons Required (x_1=25,x_2=20)**	**Tons Available**	**Unused Tons**
Material 1	(2/5)(25)+(1/2)(20)=20	20	0
Material 2	0(25)+(1/5)(20)=4	5	1
Material 3	(3/5)(25)+(3/10)(20)=21	21	0

Thus, the complete solution tells management that the optimal solution will require all available material 1 and material 3, but only 4 of the 5 tons of material 2. The 1 ton of material 2 is referred to as *slack*.

Χαλαρότητα

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Standard Form (Τυπική Μορφή)

Χαλαρές Μεταβλητές

Slack variables are added to represent the slack or idle capacity

After the addition of *slack variables*, the mathematical model of the RMC problem appears as follows:

$$\max 40x_1 + 30x_2 + 0s_1 + 0s_2 + 0s_3$$

s.t.

$$\begin{aligned} 2/5x_1 + 1/2x_2 + 1s_1 \qquad\qquad &= 20 \\ 1/5x_2 \qquad + 1s_2 \qquad &= 5 \\ 3/5x_1 + 3/10x_2 \qquad\qquad + 1s_3 &= 21 \end{aligned}$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

Whenever a linear program is written with all the constraints expressed as equalities, it is said to be written in <u>*standard form*</u>.

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Par, Inc., is a small manufacturer of golf equipment and supplies whose management has decided to move into the market for medium- and high-priced golf bags. Par's distributor is enthusiastic about the new product line and has agreed to buy all the golf bags Par produces over the next 3 months.

After a thorough investigation of the steps involved in manufacturing a golf bag, management has determined that each golf bag produced will require the following operations:

1. Cutting and dyeing the material
2. Sewing
3. Finishing (inserting umbrella holder, club separators, etc.)
4. Inspection and packaging.

The director of manufacturing has analyzed each of the operations and concluded that if the company produces a medium-priced standard model, each bag will require 7/10 hour in the cutting and dyeing department, 1/2 hour in the sewing department, 1 hour in the finishing department, and 1/10 hour in the inspection and packaging department. The more expensive deluxe model will require 1 hour for cutting and dyeing, 5/6 hour for sewing, 2/3 hour for finishing, and 1/4 hour for inspection and packaging.

This production information is summarized in the table below.

The accounting department has analysed these production figures, assigned all relevant variable costs, and arrived at prices for both bags that will result in a profit contribution[1] of $10 for every standard bag and $9 for every deluxe bag produced.

In addition, after studying departmental work load projections, the director of manufacturing estimates that 630 hours for cutting and dyeing, 600 hours for sewing, 708 hours for finishing, and 135 hours for inspection and packaging will be available for the production of golf bags during the next 3 months.

[1]From an accounting perspective, this is more correctly described as the contribution margin per bag: for example, overhead and other shared costs have not been allocated.

	Production Time (hours)			
Product	Cutting and Dyeing	Sewing	Finishing	Inspection and Packaging
Standard bag	7/10	1/2	1	1/10
Deluxe bag	1	5/6	2/3	1/4

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

$$\max \quad 10x_1 + 9x_2$$

subject to (s.t)

$$7/10\, x_1 + 1x_2 \le 630 \qquad \text{Cutting and dyeing}$$

$$1/2\, x_1 + 5/6\, x_2 \le 600 \qquad \text{Sewing}$$

$$1x_1 + 2/3\, x_2 \le 708 \qquad \text{Finishing}$$

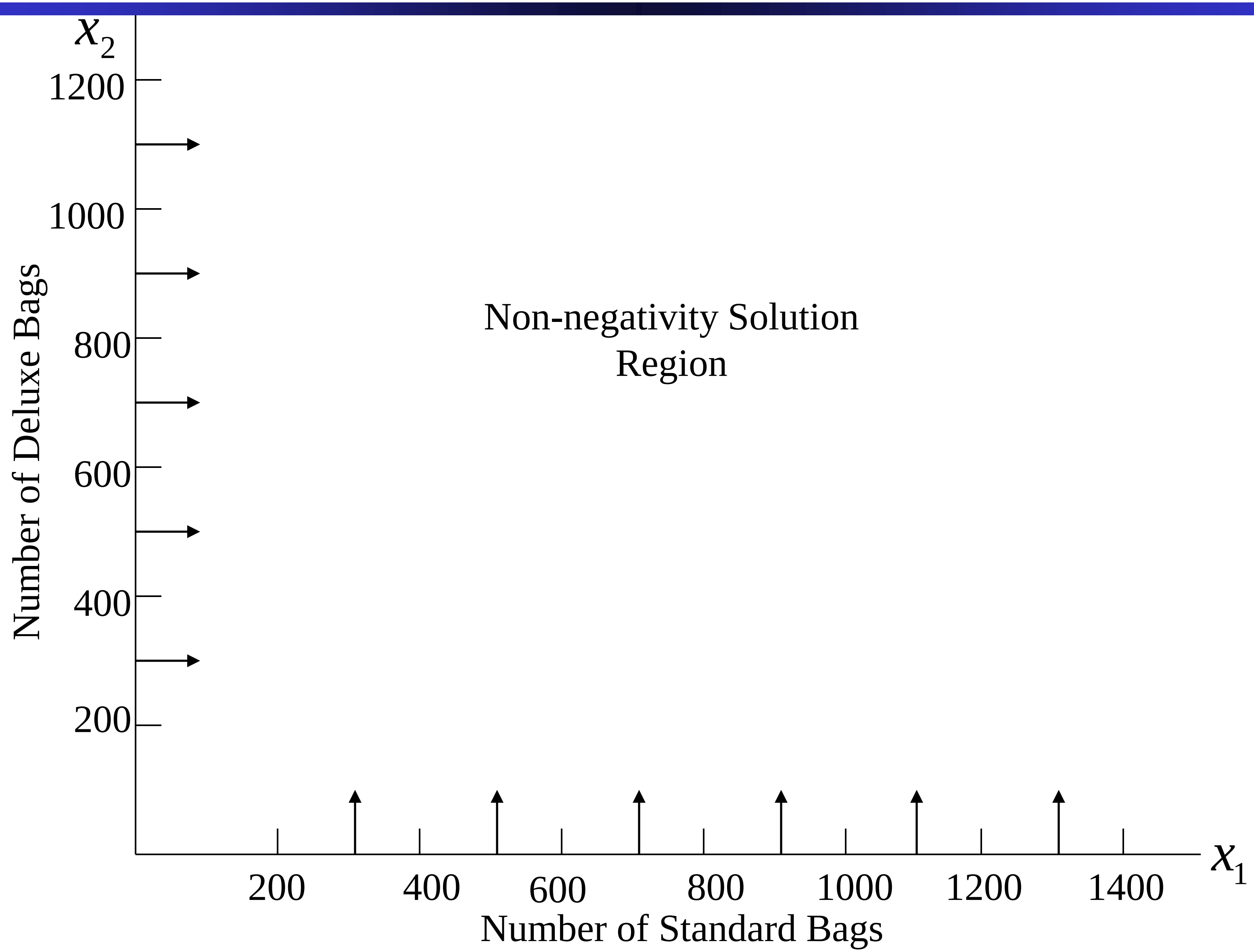
$$1/10\, x_1 + 1/4\, x_2 \le 135 \qquad \text{Inspection and packaging}$$

$$x_1, x_2 \ge 0$$

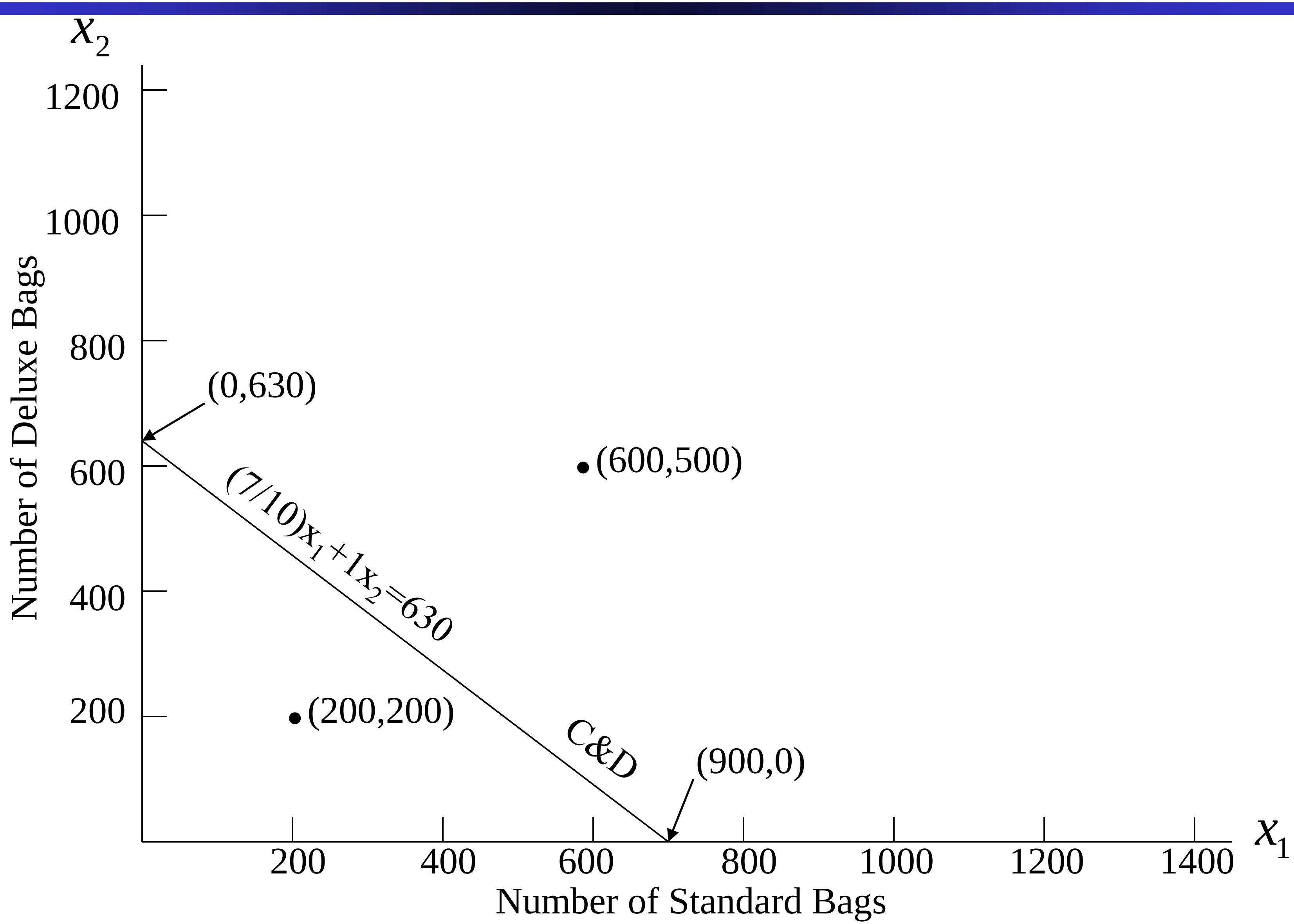
x_1 = number of standard bags Par, Inc., produces

x_2 = number of deluxe bags Par, Inc., produces

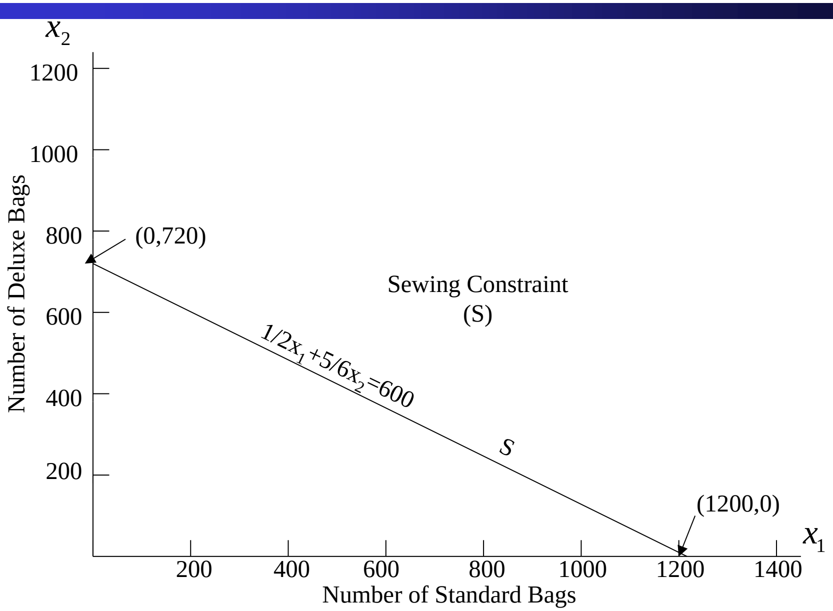
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

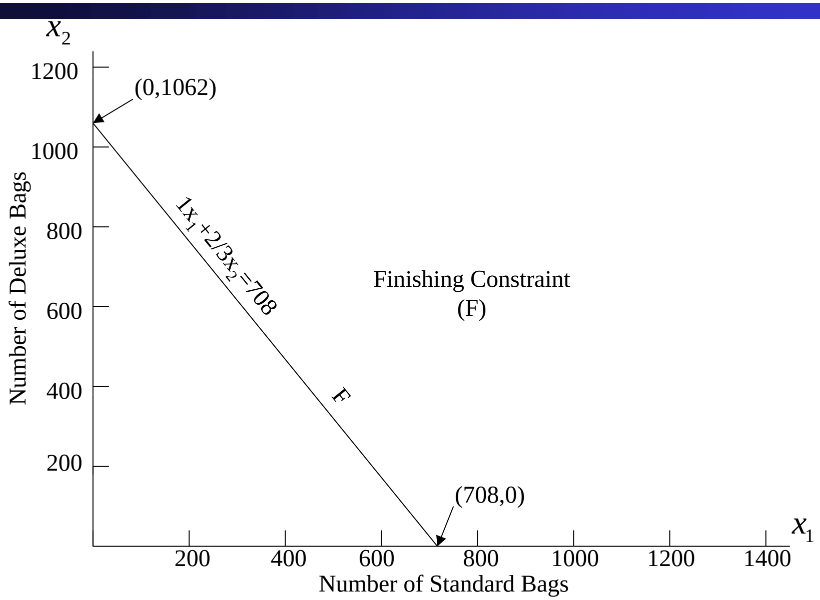


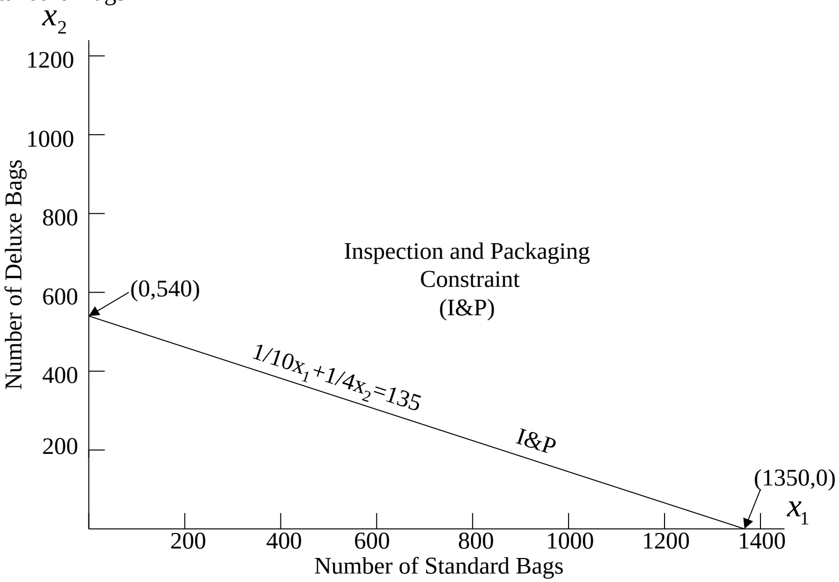
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



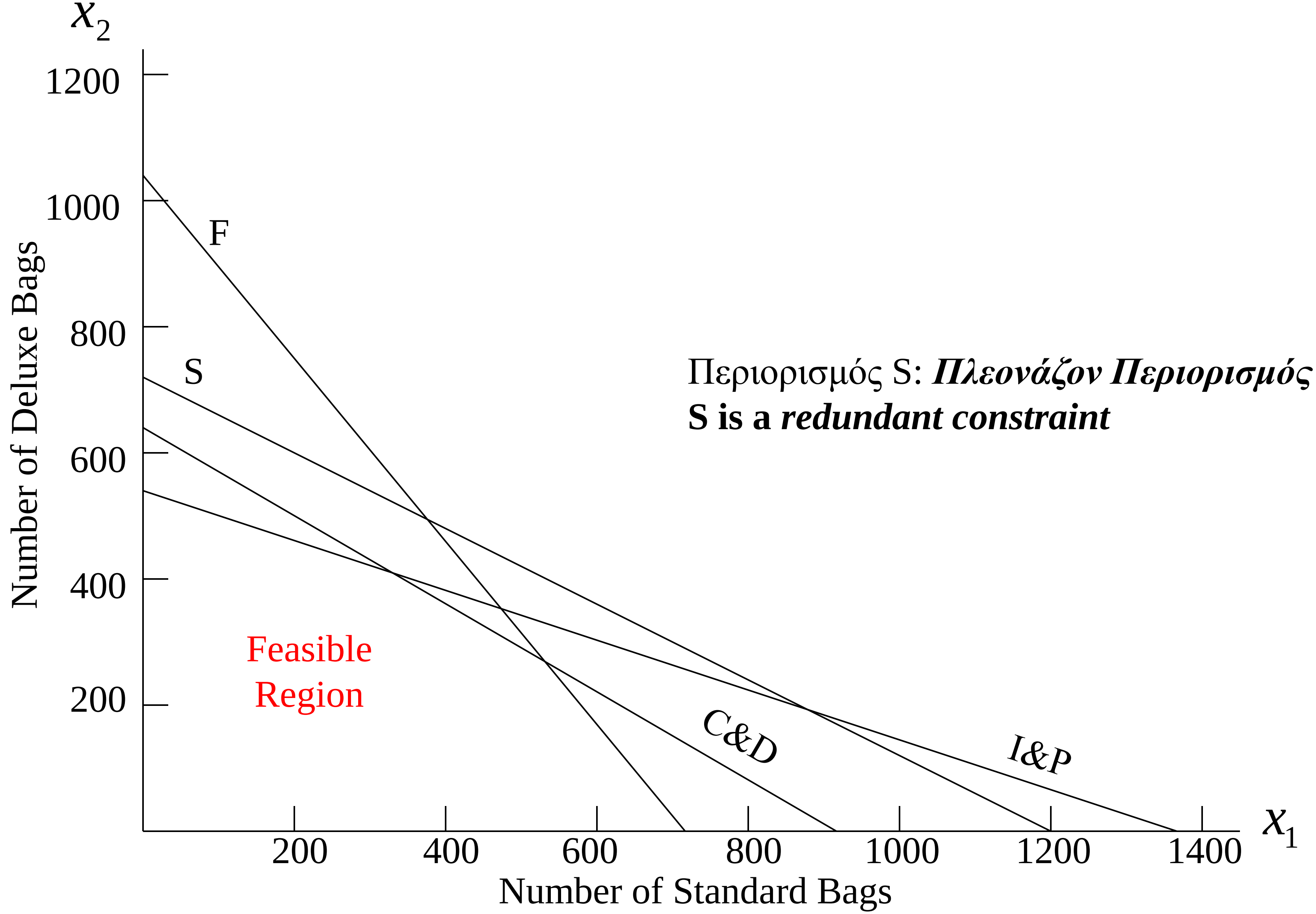
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs







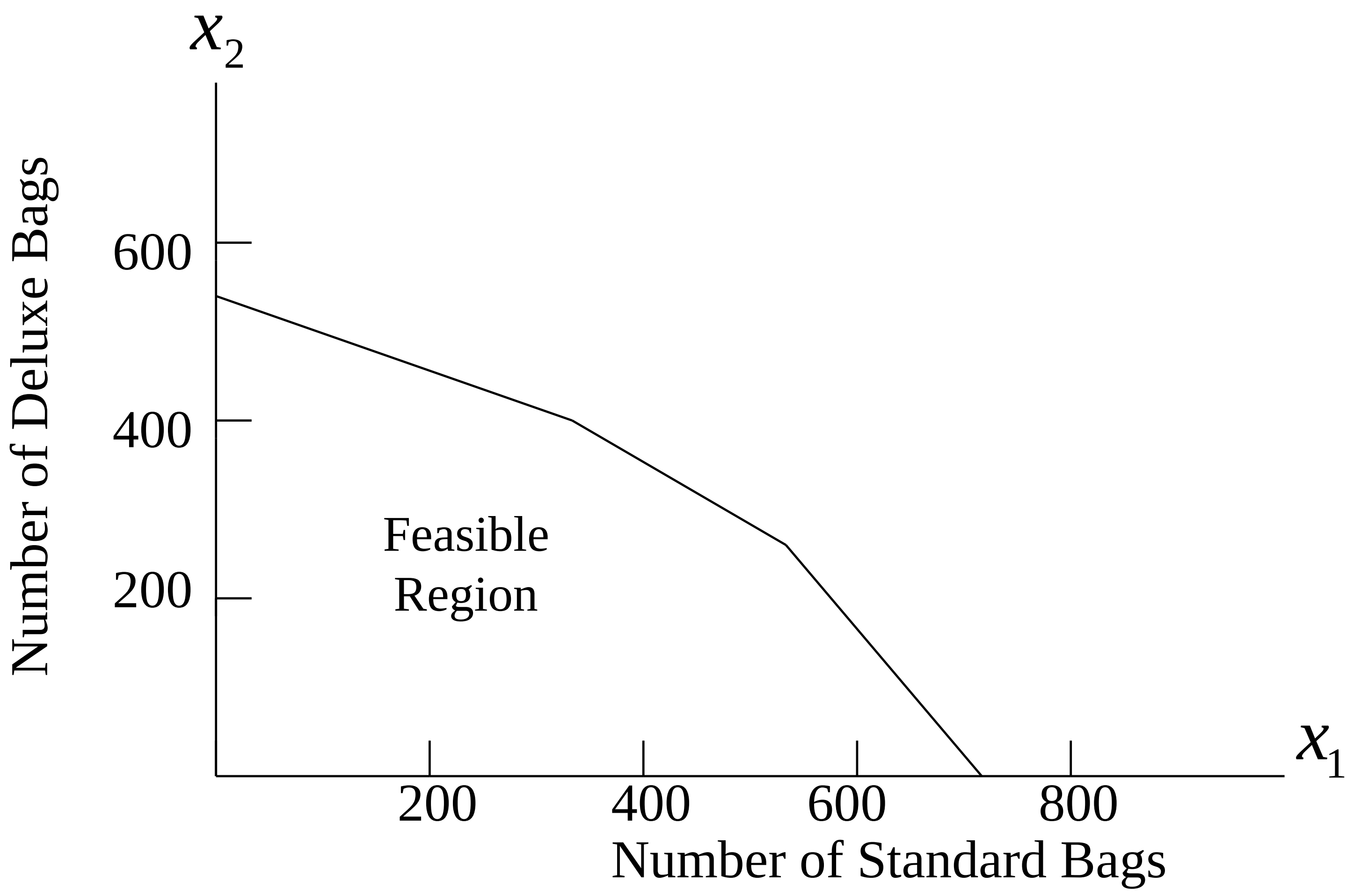
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



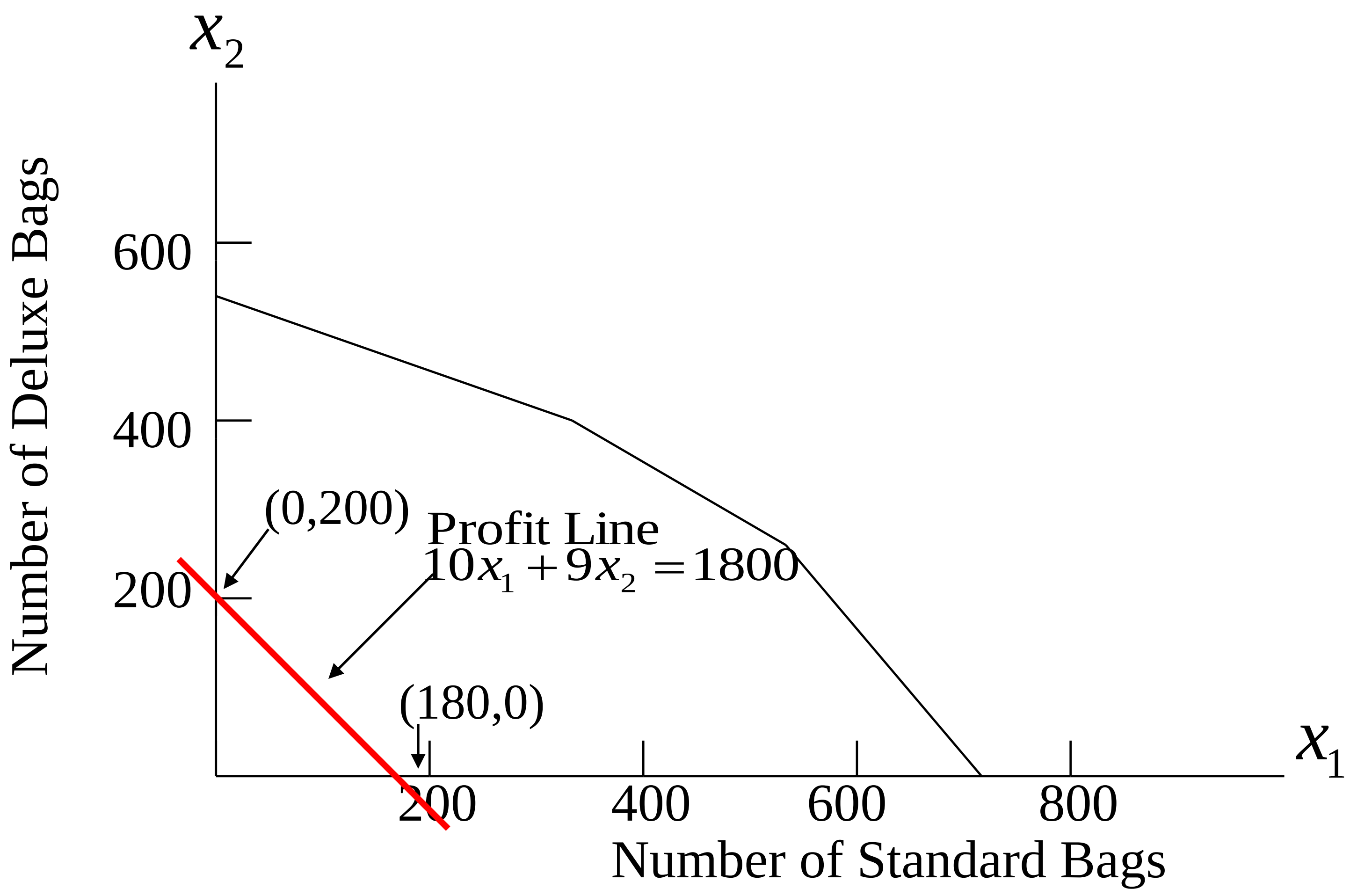
Περιορισμός S: ***Πλεονάζον Περιορισμός***

S is a *redundant constraint*

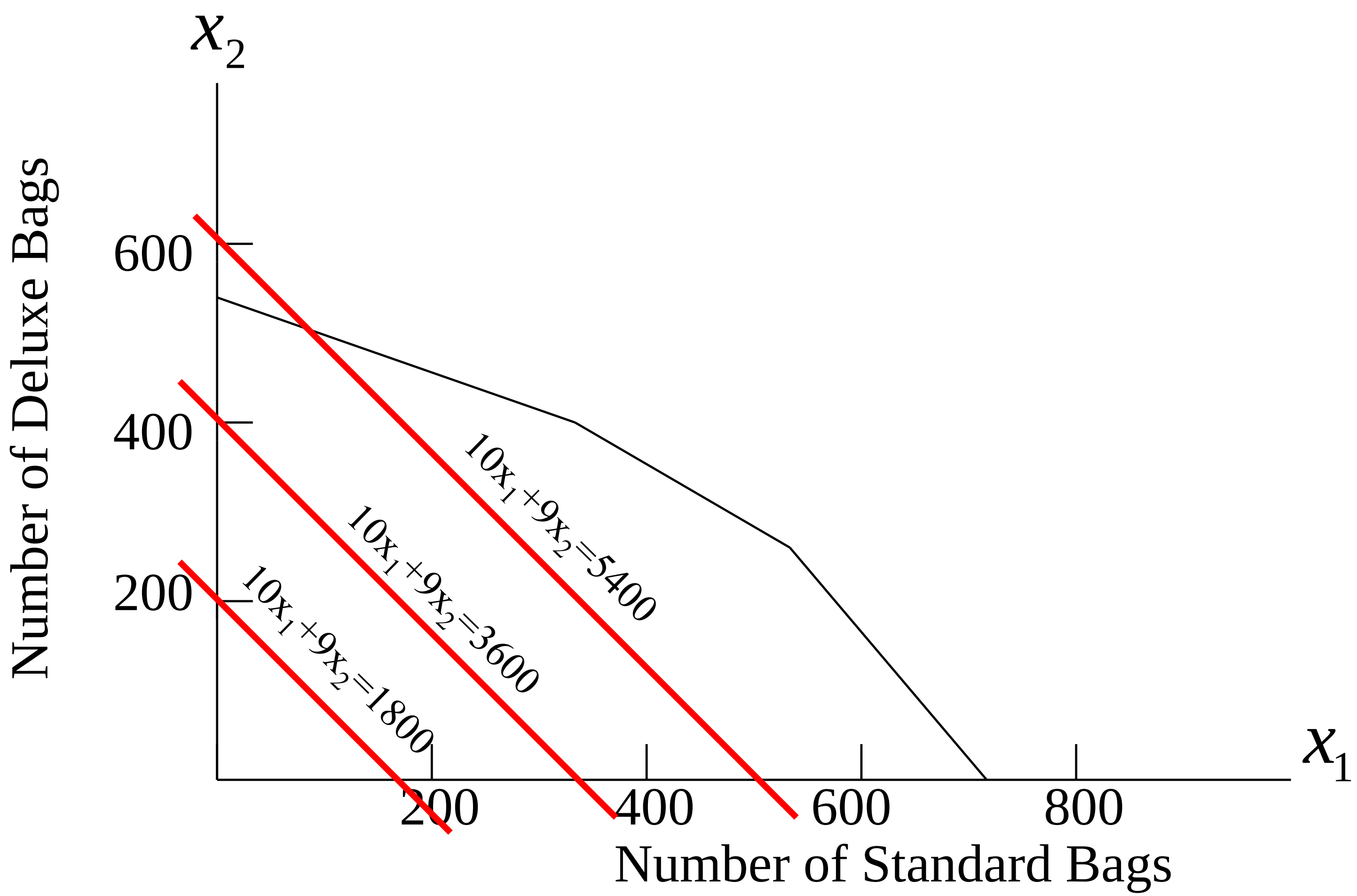
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



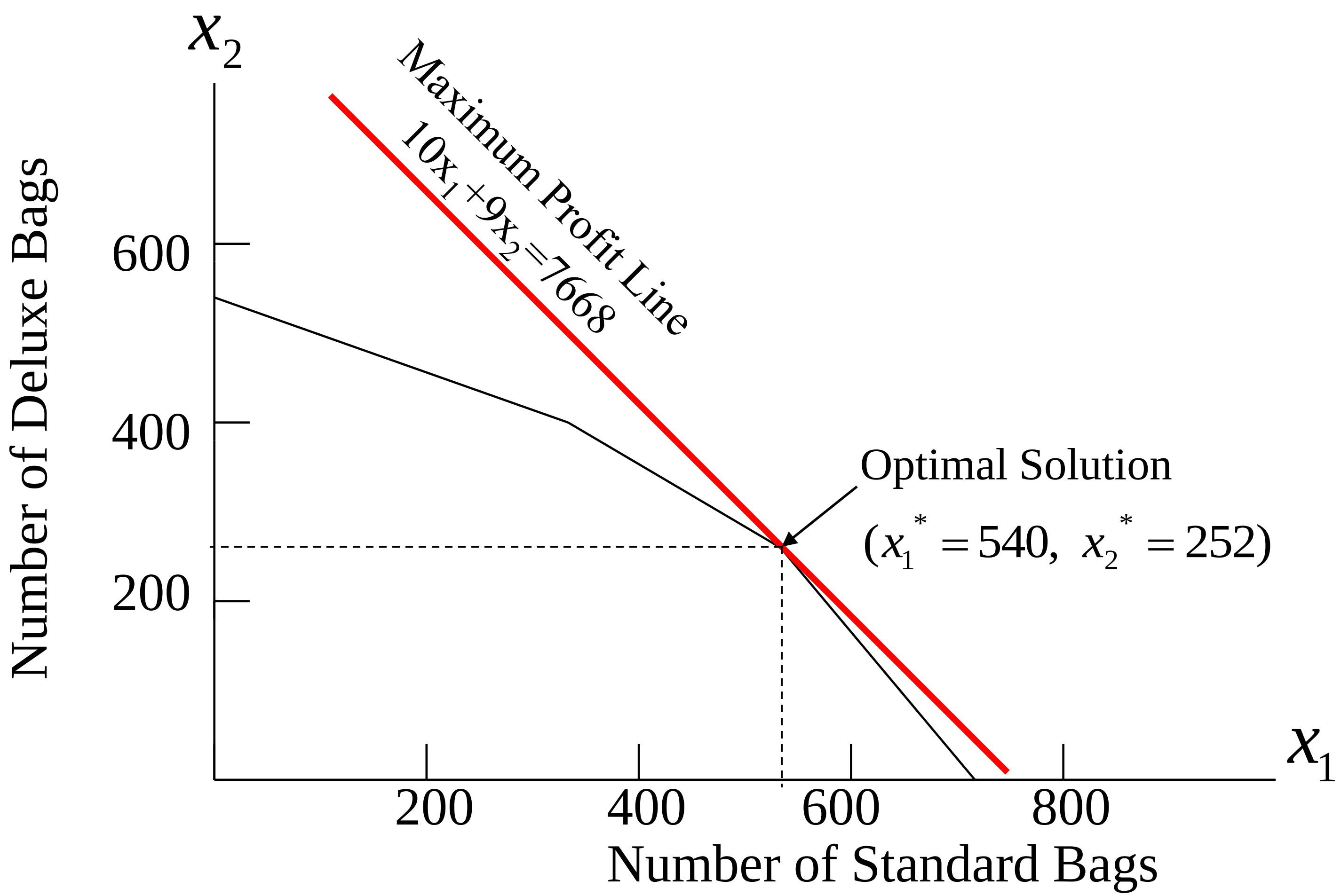
ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Ενεργοί Περιορισμοί -- Binding Constraints:

1) Cutting & Dyeing: $\frac{7}{10}x_1 + x_2 \leq 630$

2) Finishing : $x_1 + \frac{2}{3}x_2 \leq 708$

Αλγεβρική εξεύρεση λύσης -- Finding the solution by solving the system of equations:

$$\left.\begin{aligned} \frac{7}{10}x_1 + x_2 &= 630 \\ x_1 + \frac{2}{3}x_2 &= 708 \end{aligned}\right\} \longrightarrow$$

Επίλυση συστήματος δύο εξισώσεων

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

$$\frac{7}{10}x_1 + x_2 = 630 \Rightarrow \frac{7}{10}x_1 = 630 - x_2$$

$$\Rightarrow x_1 = 900 - \frac{10}{7}x_2$$

Substituting $x_1 = 900 - \frac{10}{7}x_2$ in $x_1 + \frac{2}{3}x_2 = 708$ gives

$$(900 - \frac{10}{7}x_2) + \frac{2}{3}x_2 = 708$$

$$900 - \frac{30}{21}x_2 + \frac{14}{21}x_2 = 708$$

$$-\frac{16}{21}x_2 = -192$$

$$x_2 = \frac{192}{(16/21)} = \underline{\underline{252}}$$

$$\Rightarrow x_1 = 900 - \frac{10}{7}(252) = \underline{\underline{540}}$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

$$\max 10x_1 + 9x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$$

s.t.

$$7/10\,x_1 + 1x_2 + 1s_1 = 630$$

$$1/2\,x_1 + 5/6\,x_2 + 1s_2 = 600$$

$$1x_1 + 2/3\,x_2 + 1s_3 = 708$$

$$1/10\,x_1 + 1/4\,x_2 + 1s_4 = 135$$

$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

Constraint	Value of Slack Variable
Cutting and dyeing	$s_1=0$
Sewing	$s_2=120$
Finishing	$s_3=0$
Inspection and packaging	$s_4=18$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The Par Inc., problem involved *maximization*; however, many linear programming problems involve *minimization*. For example, consider the case of M&D Chemicals. M&D Chemicals produces two products that are sold as raw materials to companies manufacturing bath soaps and laundry detergents.

Based on an analysis of current inventory levels and potential demand for the coming month, M&D's management has specified that the combined production for products 1 and 2 must total at least 350 gallons. Separately, a major customer's order for 125 gallons of product 1 must also be satisfied. Product 1 requires 2 hours of processing time per gallon while product 2 requires 1 hour of processing time per gallon, and for the coming month, 600 hours of processing time are available. M&D's objective is to satisfy the above requirements at a *minimum total production cost*. Production costs are $2 per gallon for product 1 and $3 per gallon for product 2.

To find the *minimum-cost production schedule*, let us write the M&D Chemicals problem as a linear program (LP). Following a procedure similar to the one used for Par, Inc., we first define the decision variables and the objective function for the problem.

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Let

x_1 = number of gallons of product 1 produced

x_2 = number of gallons of product 2 produced

$$\min \; 2x_1 + 3x_2$$

s.t.

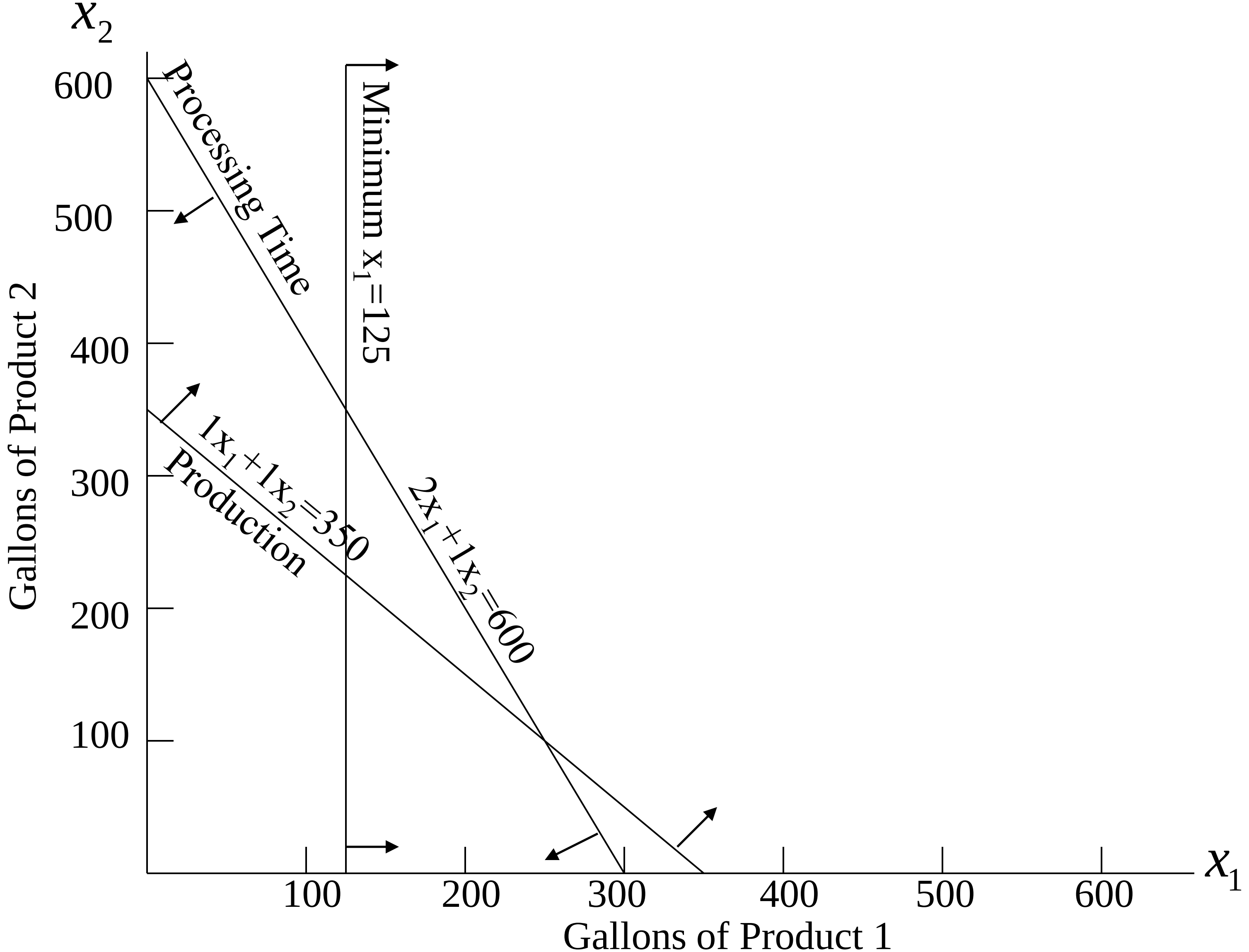
$$1x_1 \geq 125 \quad \text{Demand for product 1}$$

$$1x_1 + 1x_2 \geq 350 \quad \text{Total production}$$

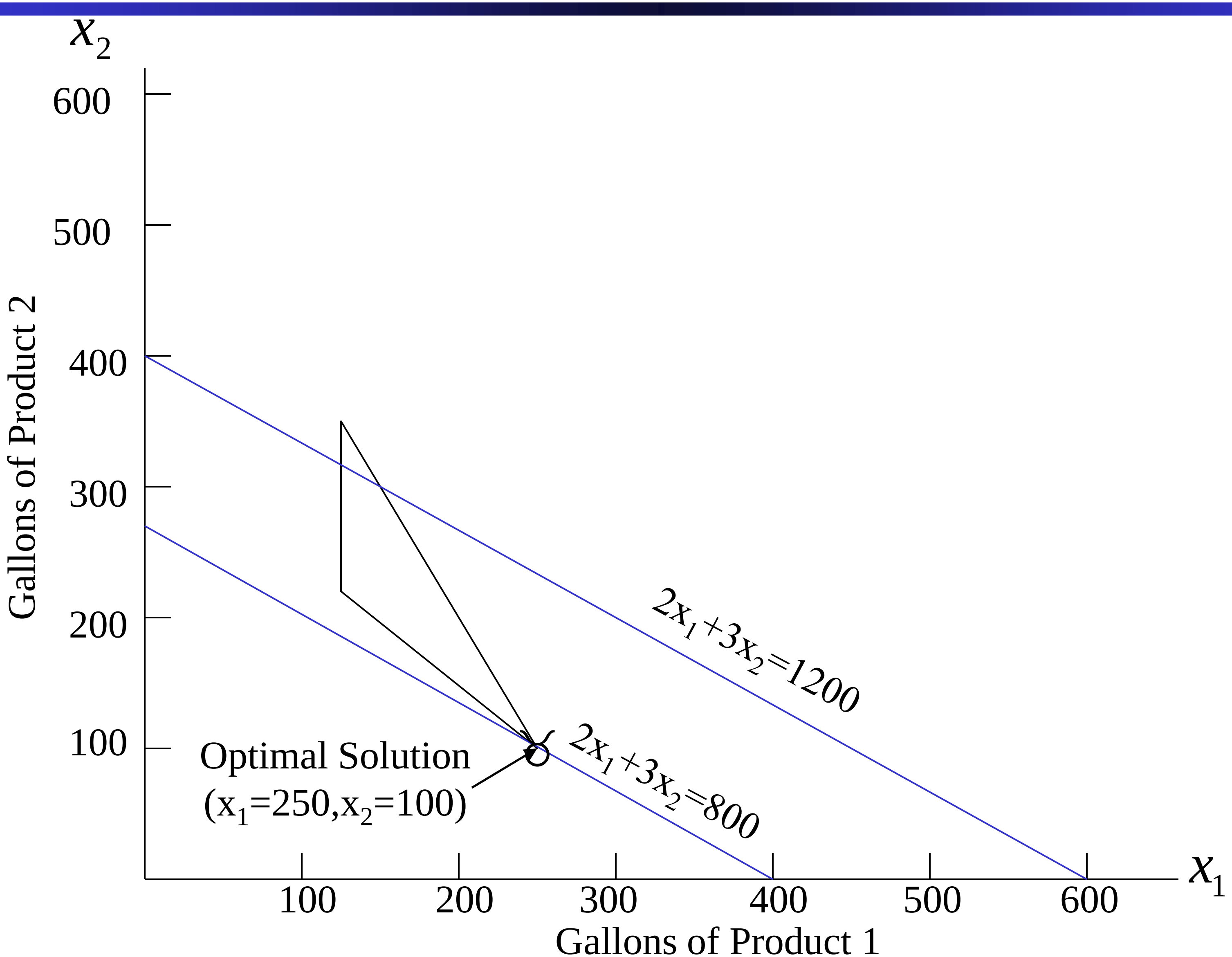
$$2x_1 + 1x_2 \leq 600 \quad \text{Processing time}$$

$$x_1, x_2 \geq 0$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs



ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

$$\text{Min } 2x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$$

s.t.

$$1x_1 \quad - 1s_1 \quad = 125$$

$$1x_1 + 1x_2 \quad - 1s_2 \quad = 350$$

$$2x_1 + 1x_2 \quad + 1s_3 = 600$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

Constraint	Value of Surplus or Slack Variables
Demand for product 1	s_1=125
Total production	s_2=0
Processing time	s_3=0

} ***Surplus Variables***
Πλεονάζουσες Μεταβλητές

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

$$\text{Min } 2x_1 + 2x_2$$

s.t.

$$1x_1 + 3x_2 \le 12$$
$$3x_1 + 1x_2 \ge 13$$
$$1x_1 - 1x_2 = 3$$
$$x_1, x_2 \ge 0$$

The standard-form representation of this problem is

$$\text{Min } 2x_1 + 2x_2 + 0s_1 + 0s_2$$

s.t.

$$1x_1 + 3x_2 + 1s_1 = 12$$
$$3x_1 + 1x_2 - 1s_2 = 13$$
$$1x_1 - 1x_2 = 3$$
$$x_1, x_2, s_1, s_2 \ge 0$$

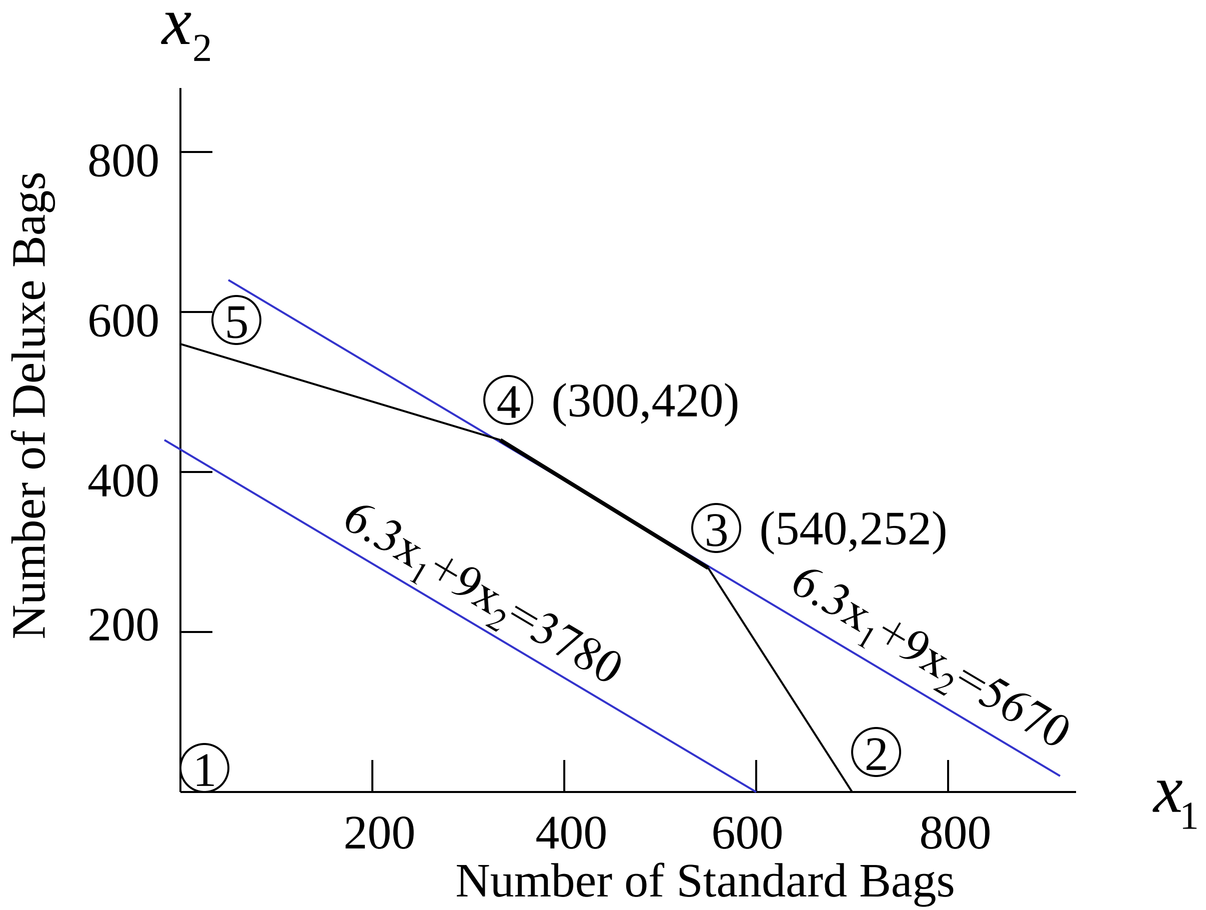
ΕΙΔΙΚΕΣ ΠΕΡΙΠΤΩΣΕΙΣ ΛΥΣΕΩΝ
SPECIAL SOLUTION CASES

- **Εναλλακτικές άριστες λύσεις, άπειρες λύσεις**
  - Alternative optimal solutions, Infinite solutions
- **Μη εφικτή λύση**
  - Infeasibility
- **Αφραγη λύση**
  - Unboundedness

ΕΝΑΛΛΑΚΤΙΚΕΣ ΛΥΣΕΙΣ
ALTERNATIVE SOLUTIONS

$$Z = 10x_1 + 9x_2 \Rightarrow Z = 6.3x_1 + 9x_2$$

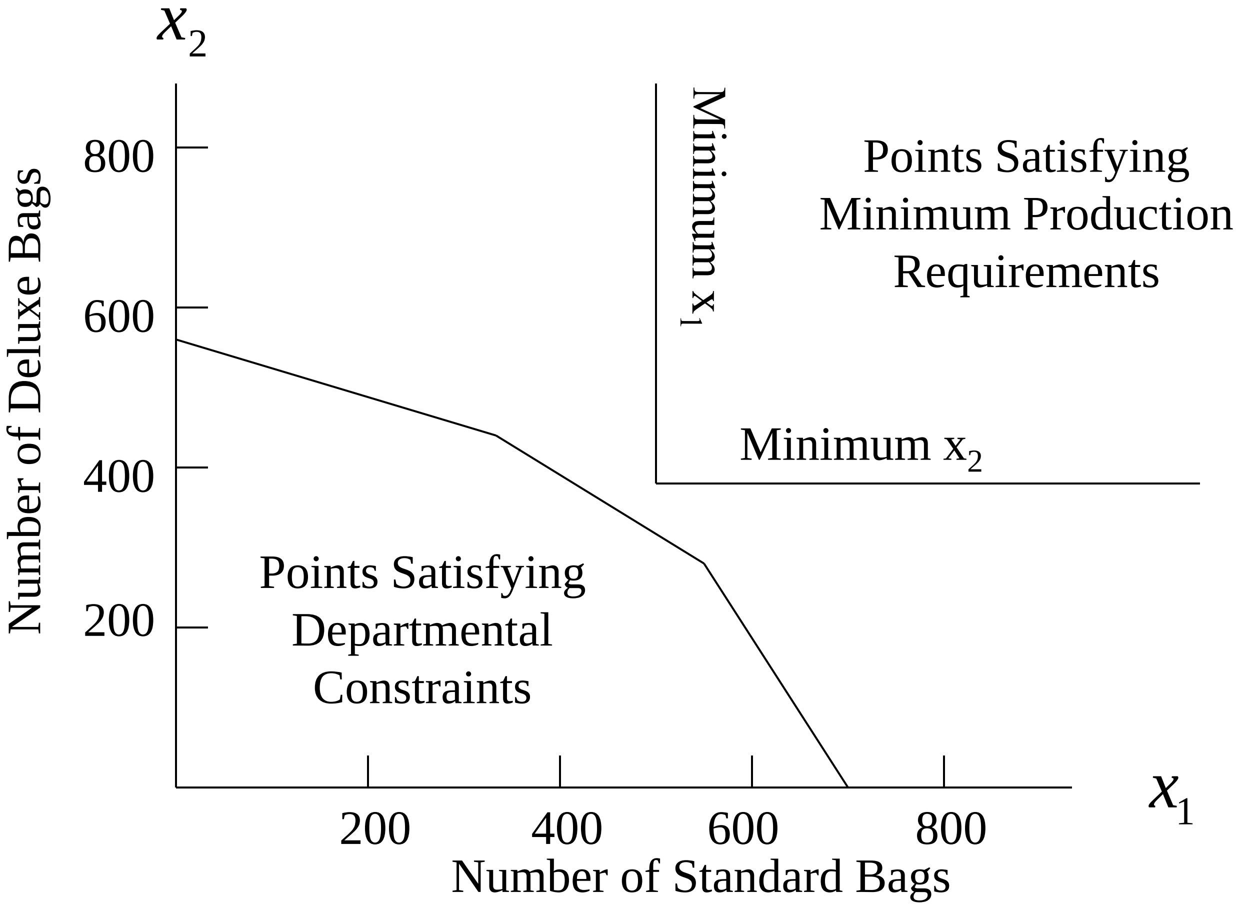
(change in profit contributions in earlier example)



ΜΗ ΕΦΙΚΤΗ ΛΥΣΗ
INFEASIBILITY

New Constraints : $x_1 \geq 500$

$x_2 \geq 360$



ΜΗ ΕΦΙΚΤΗ ΛΥΣΗ
INFEASIBILITY

Operation	Minimum Required Resources (hours)	Available Resources (hours)	Additional Resources Needed (hours)
Cutting and dyeing	(7/10)(500)+1(360)=710	630	80
Sewing	(1/2)(500)+(5/6)(360)=550	600	None
Finishing	1(500)+(2/3)(360)=740	708	32
Inspection and packaging	(1/10)(500)+(1/4)(360)=140	135	5

ΑΦΡΑΓΗ ΛΥΣΗ
UNBOUNDEDNESS

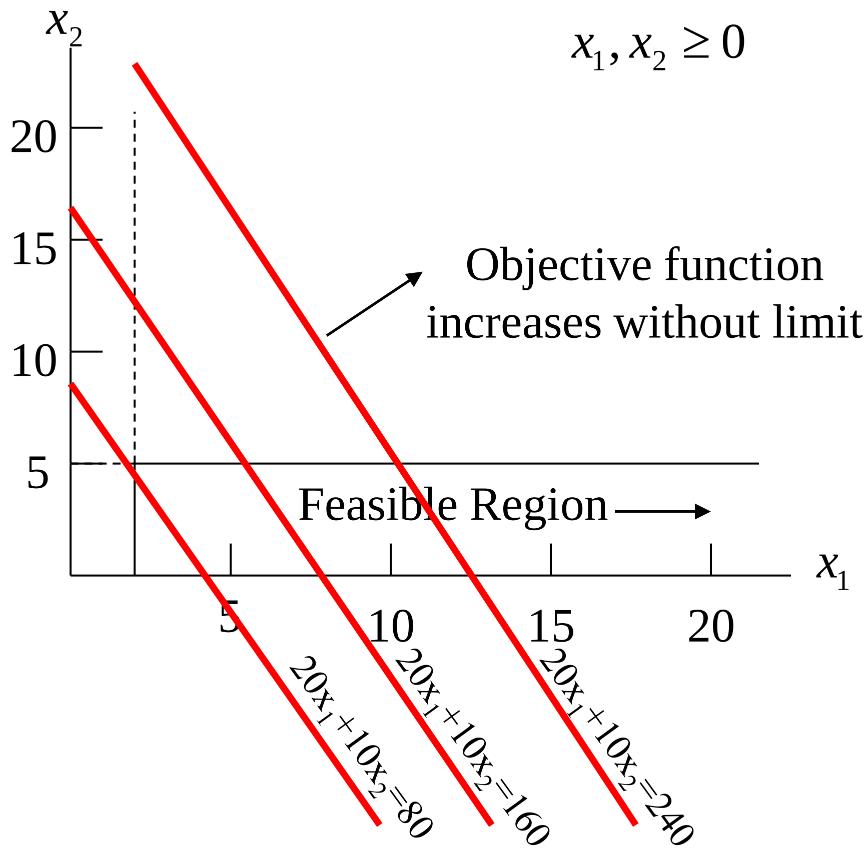
$$\max 20x_1 + 10x_2$$

s.t.

$$1x_1 \geq 2$$

$$1x_2 \leq 5$$

$$x_1, x_2 \geq 0$$



ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

- **Πως μια αλλαγή σε συντελεστή της αντικειμενικής συνάρτησης επηρεάζει τη βέλτιστη λύση;**
  - How will a change in a coefficient of the objective function affect the optimal solution?

- **Πως μια αλλαγή στην τιμή της δεξιάς πλευράς περιορισμού επηρεάζει τη βέλτιστη λύση;**
  - How will a change in the right-hand side for a constraint affect the optimal solution?

ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

The mathematical statement or formulation of the RMC problem is written as follows:

$$\max \quad 40x_1 + 30x_2$$

s.t.

$$\frac{2}{5}x_1 + \frac{1}{2}x_2 \le 20 \quad \text{Material 1}$$

$$\frac{1}{5}x_2 \le 5 \quad \text{Material 2}$$

$$\frac{3}{5}x_1 + \frac{3}{10}x_2 \le 21 \quad \text{Material 3}$$

$$x_1, x_2 \ge 0$$

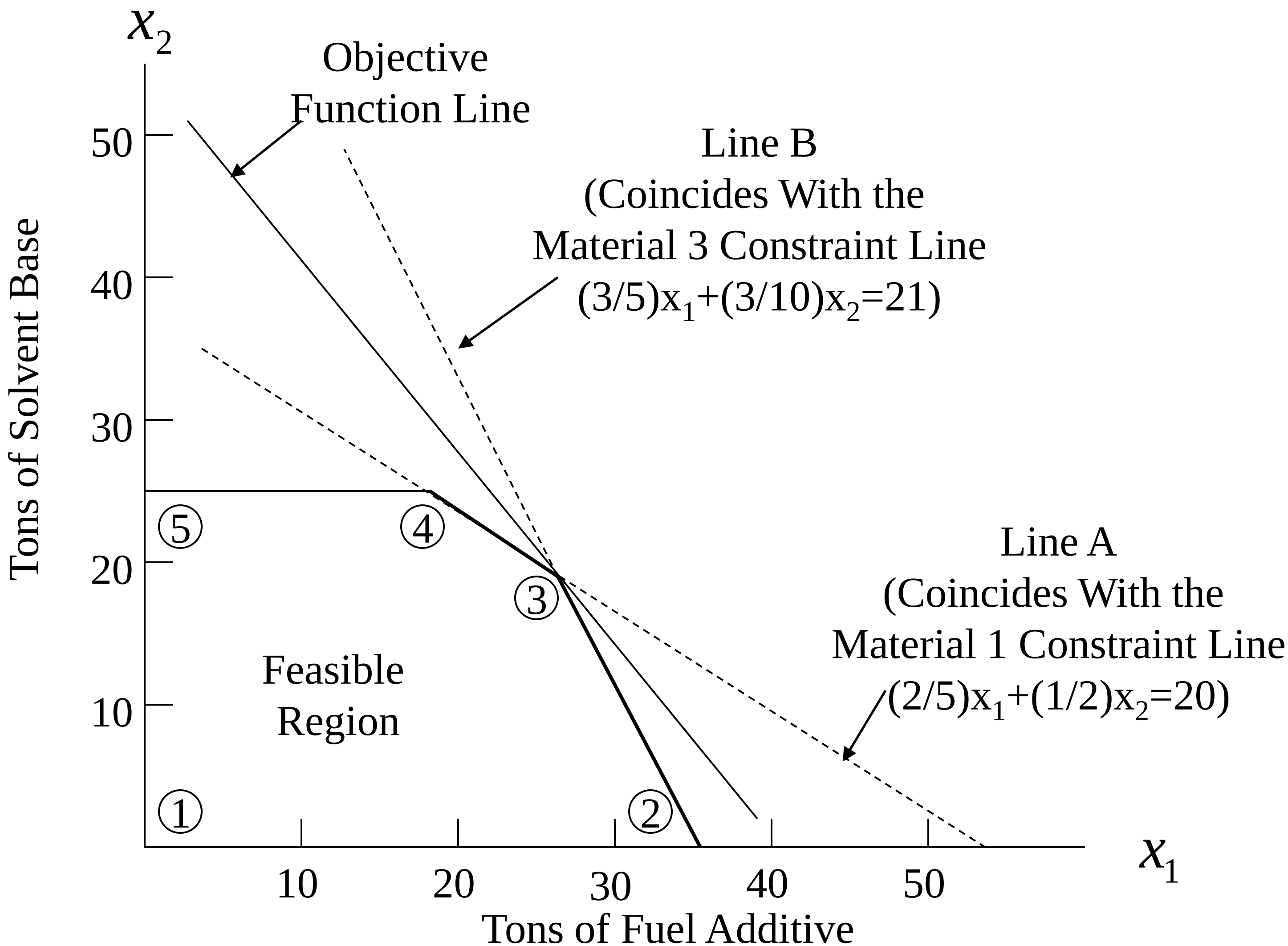
where

x_1 = the number of tons of fuel additive that RMC produces

x_2 = the number of tons of solvent base that RMC produces

ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

Sensitivity Analysis: Changes in Objective Function Coefficients



ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

Extreme point 3 will be optimal as long as

Slope of line B ≤ Slope of objective function line ≤ Slope of line A

The equation for line A in its slope-intercept form is

$$\tfrac{1}{2} x_2 = -\tfrac{2}{5} x_1 + 20$$

$$x_2 = -\tfrac{4}{5} x_1 + 40$$

The equation for line B in its slope-intercept form is

$$\tfrac{3}{10} x_2 = -\tfrac{3}{5} x_1 + 21$$

$$x_2 = -2x_1 + 70$$

Therefore, for extreme point 3 to be optimal we must have

$$-2 \leq \text{Slope of the objective function line} \leq -\tfrac{4}{5}$$

ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

The slope-intercept form for the objective function line is

$$x_2 = -\frac{c_1}{c_2}x_1 + \frac{z}{c_2}$$

Thus, extreme point 3 will be optimal as long as

$$-2 \le -\frac{c_1}{c_2} \le -\frac{4}{5}$$

Αριστο Διάστημα

Computing the Range of Optimality for the Fuel Additive Coefficient

$$-2 \le -\frac{c_1}{30} \le -\frac{4}{5}$$

$$-2 \le -\frac{c_1}{30} \qquad\qquad -\frac{c_1}{30} \le -\frac{4}{5}$$

$$-60 \le -c_1 \text{ or } c_1 \le 60 \qquad\qquad -c_1 \le -\frac{120}{5} \le -24 \text{ or } c_1 \ge 24$$

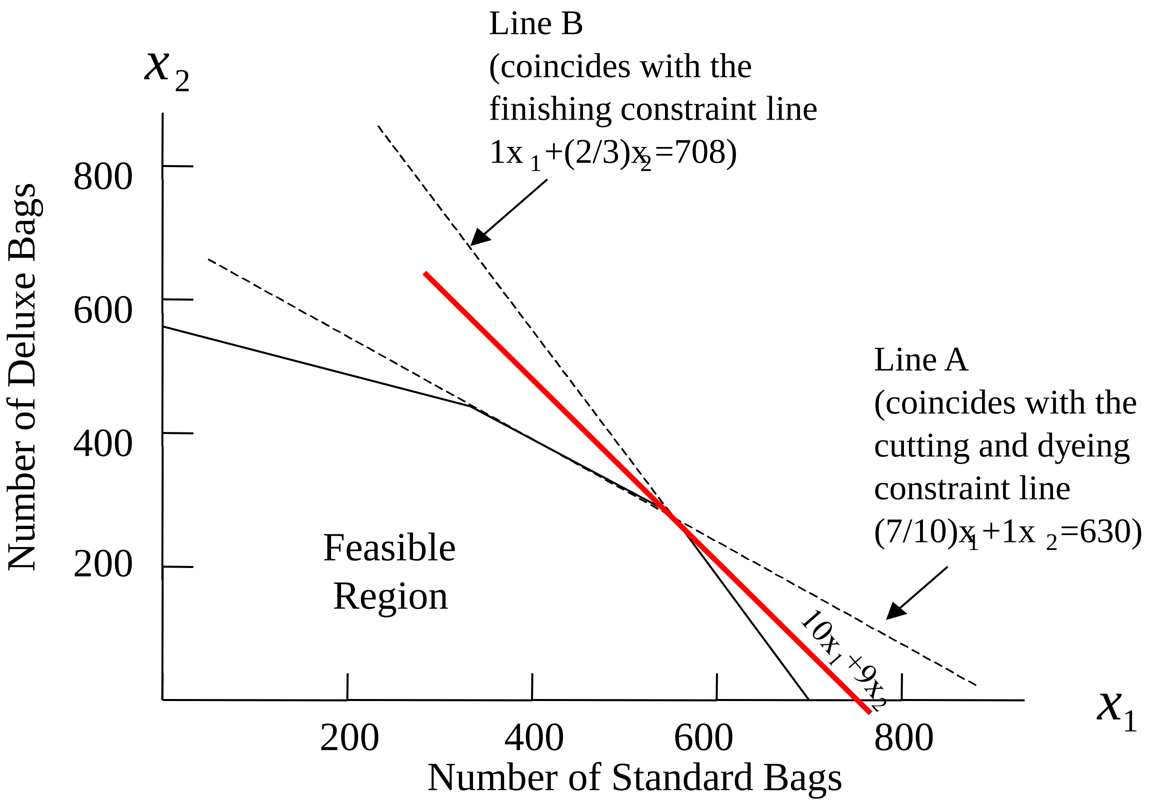
$$\mathbf{24 \le c_1 \le 60}$$

ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

$$\max \quad 10x_1 + 9x_2$$

subject to (s.t)

$$7/10\,x_1 + 1x_2 \le 630 \quad \text{Cutting and dyeing}$$
$$1/2\,x_1 + 5/6\,x_2 \le 600 \quad \text{Sewing}$$
$$1x_1 + 2/3\,x_2 \le 708 \quad \text{Finishing}$$
$$1/10\,x_1 + 1/4\,x_2 \le 138 \quad \text{Inspection and packaging}$$
$$x_1, x_2 \ge 0$$



ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

$$\frac{7}{10}x_1 + x_2 = 630 \Rightarrow x_2 = -\frac{7}{10}x_1 + 630$$

slope

$$x_1 + \frac{2}{3}x_2 = 708 \Rightarrow x_2 = -\frac{3}{2}x_1 + 1062$$

slope

$$-\frac{3}{2} \leq \text{slope of objective function} \leq -\frac{7}{10}$$

$$Z = c_1x_1 + c_2x_2 \Rightarrow x_2 = -\frac{c_1}{c_2}x_1 + \frac{Z}{c_2}$$

slope of objective function

$$-\frac{3}{2} \leq -\frac{c_1}{c_2} \leq -\frac{7}{10}$$

$$\text{if } \; c_2 = 9 \Rightarrow -\frac{27}{2} \leq -c_1 \leq -\frac{63}{10}$$

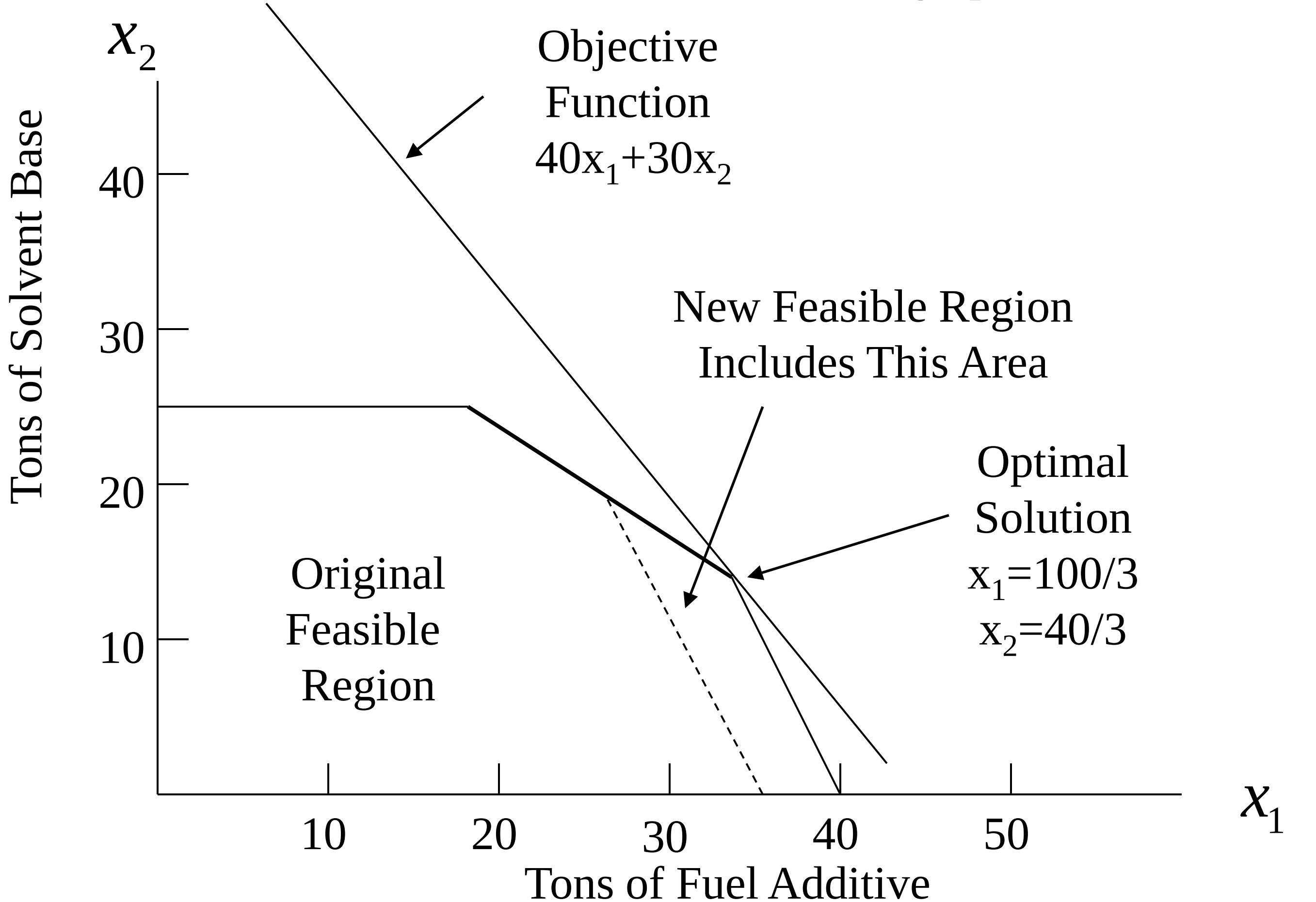
$$\Rightarrow 13.5 \geq c_1 \geq 6.3$$

Άριστο Διάστημα

range of optimality

ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

Let us consider how a change in the right-hand side value of a constraint may affect the feasible region. For example, consider what happens if an additional 3 tons of material 3 become available. Consider the new graphical solution shown below.



ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS

The new optimal solution is

$$x_1 = 100/3$$
$$x_2 = 40/3$$

The new value for the objective function is

40(100/3) + 30(40/3) = 1733.33

Since the value of the optimal solution to the original problem is $1600, increasing the right-hand side of the material 3 constraint by 3 tons provides an increase in profit of

$1733.33 - 1600 = $133.33

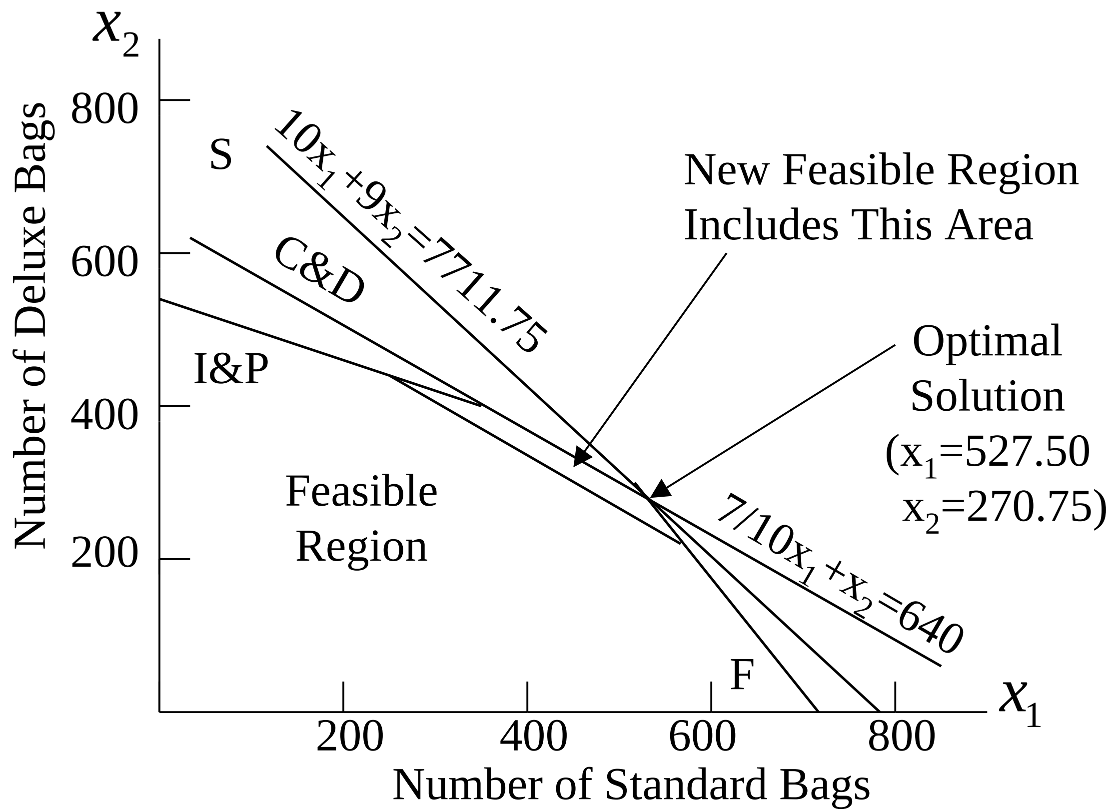
Thus, the increase in profit occurs at a rate of

Δυϊκή Τιμή: Dual Price

$133.33/3 = $44.44 per ton

$44.44 is the *dual price* for the material 3 constraint

ΑΝΑΛΥΣΗ ΕΥΑΙΣΘΗΣΙΑΣ
SENSITIVITY ANALYSIS



$$\frac{7}{10}x_1 + x_2 \leq 630 \Rightarrow \frac{7}{10}x_1 + x_2 \leq 640$$

$Z = 7668$ $\qquad$ $Z = 7711.75$

$$7711.75 - 7668 = 43.75$$

$$43.75/10 = 4.375 \longrightarrow \text{dual price}$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Solve the following linear program:

$$\text{Max } 5x_1 + 5x_2$$

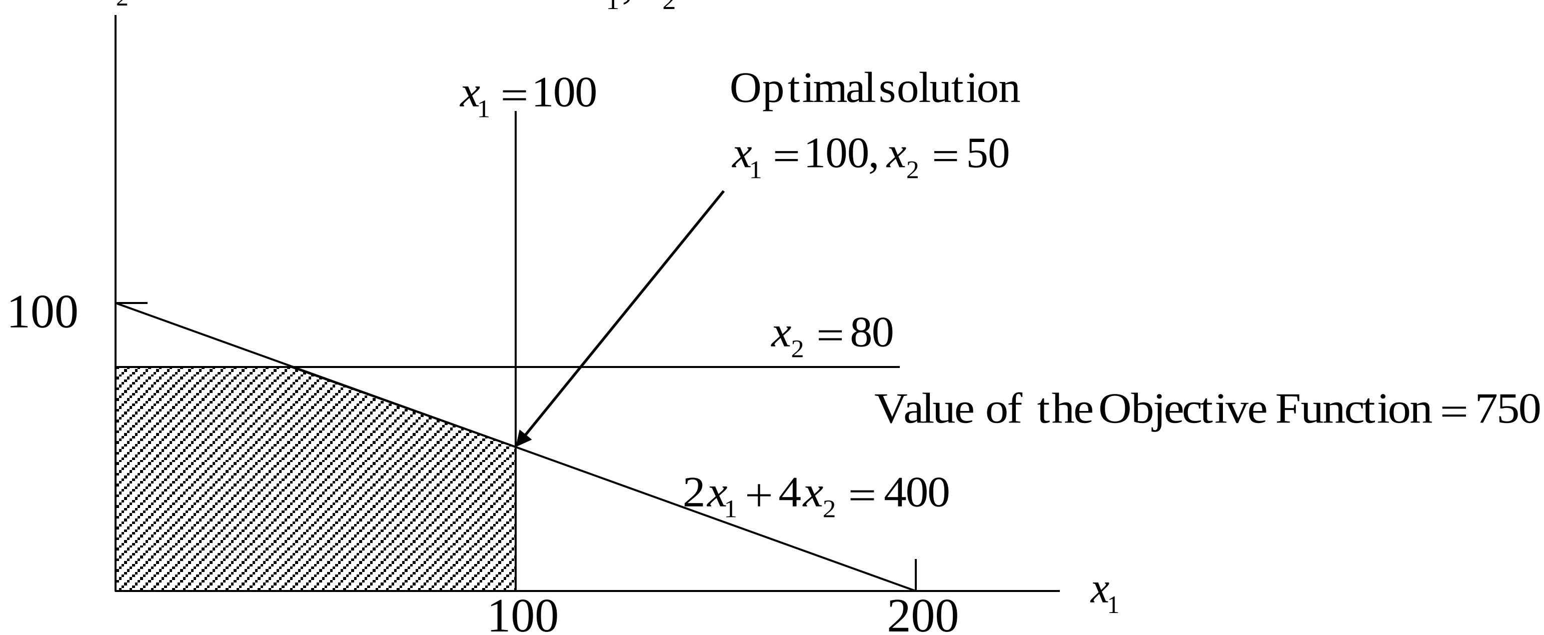
s.t.

$$1x_1 \leq 100$$

$$1x_2 \leq 80$$

$$2x_2 + 4x_2 \leq 400$$

$$x_1, x_2 \geq 0$$



ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

The Erlanger Manufacturing Company makes two products. The profit estimates are $25 for each unit of product 1 sold and $30 for each unit of product 2 sold. The labor-hour requirements for the products in each of three production departments are summarized below:

Department	Product 1	Product 2
A	1.50	3.00
B	2.00	1.00
C	.25	.25

The production supervisors in the departments have estimated that the following number of labor-hours will be available during the next month: 450 hours in department A, 350 hours in department B, and 50 hours in department C. Assuming that the company is interested in maximizing the total profit contribution, answer the following:

a. What is the linear programming model for this problem?

b. Find the optimal solution. How much of each product should be produced and what is the projected total profit contribution?

c. What is the scheduled production time and slack time in each department?

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

a. Let $x_1 = \text{units of product 1}$

$x_2 = \text{units of product 2}$

$$\text{Max } 25x_1 + 30x_2$$

s.t.

$$1.5x_1 + 3x_2 \le 450 \quad \text{Dept.A}$$

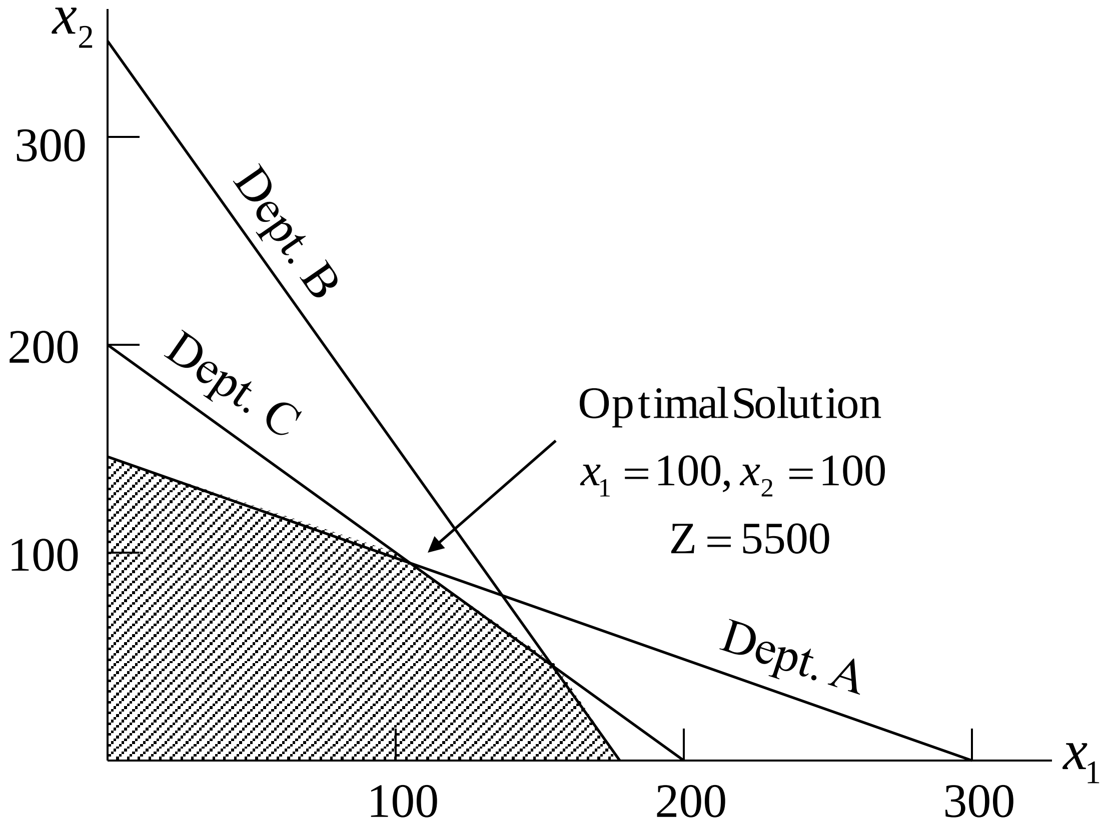
$$2x_1 + 1x_2 \le 350 \quad \text{Dept.B}$$

$$.25x_1 + .25x_2 \le 50 \quad \text{Dept.C}$$

$$x_1, x_2 \ge 0$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

b.



c.

Department	Capacity	Usage	Slack
Dept. A	450	450	0 hours
Dept. B	350	300	50 hours
Dept. C	50	50	0 hours

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Ryland Farms in northwestern Indiana grows soybeans and corn on its 500 acres of land. An acre of soybeans brings a $100 profit, and an acre of corn brings a $200 profit. Because of a government program, no more than 200 acres may be planned in soybeans. During the planting season, 1200 hours of planting time will be available. Each acre of soybeans requires 2 hours, while each acre of corn requires 6 hours. How many acres of soybeans and how many acres of corn should be planted to maximize profits?

a. What is the linear programming model for this problem?

b. Write the linear program standard form.

c. Find the optimal solution.

d. What are the values and interpretations of all slack and surplus variables?

e. If the farm could get either more hours of labor for planting or additional land, which should it attempt to obtain? Why?

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

a. Let $x_1 =$ acres of soybean

$x_2 =$ acres of corn

$$\text{Max } 100x_1 + 200x_2$$

s.t.

$$\begin{aligned} x_1 + \quad x_2 &\leq 500 \quad \text{Land} \\ x_1 \qquad\quad &\leq 200 \quad \text{Soybean limit} \\ 2x_1 + \quad 6x_2 &\leq 1200 \quad \text{Labor hours} \\ x_1, x_2 &\geq 0 \end{aligned}$$

b.

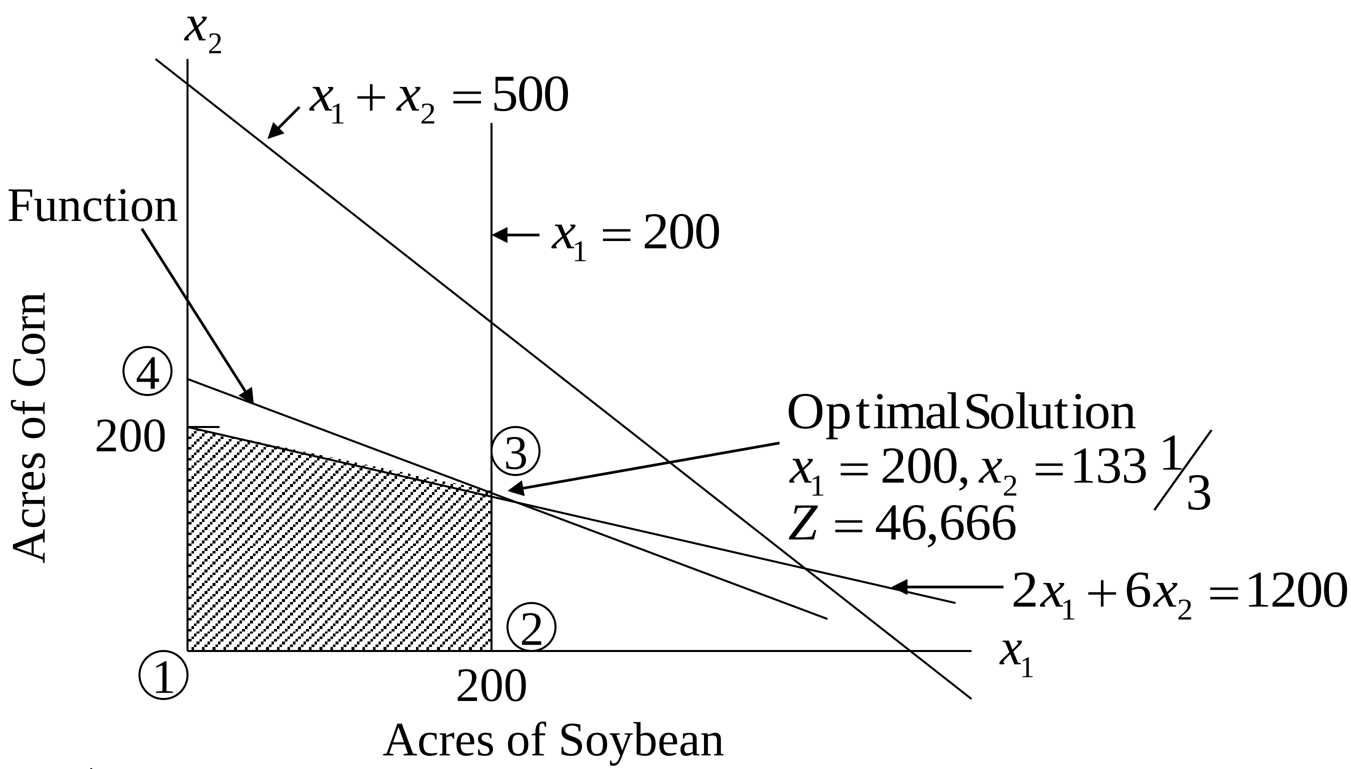
$$\text{Max } 100x_1 + 200x_2 + 0s_1 + 0s_2 + 0s_3$$

s.t.

$$\begin{aligned} x_1 + \quad x_2 + s_1 \qquad\qquad &= 500 \\ x_1 \qquad\qquad + s_2 \qquad &= 200 \\ 2x_1 + \quad 6x_2 \qquad\qquad + s_3 &= 1200 \\ x_1, x_2, s_1, s_2, s_3 &\geq 0 \end{aligned}$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

c.



d. $s_1 = 166\frac{2}{3}$ = unused land

$s_2 = 0$ = amount below the soybean limit of 200 acres

$s_3 = 0$ = unused planting time

e. Binding constraints: soybean limit and labor hours

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Car Phones, Inc., sells two models of car telephones: model x and model y. Records show that 3 hours of sales time are used for each model x phone that is sold and 5 hours of sales time for each model y phone. A total of 600 hours of sales time is available for the next 4-week period. In addition, management planning policies call for minimum sales goals of 25 units for both model x and model y.

a. Show the feasible region for the Car Phones, Inc., problem.

b. Assuming the company makes a $40 profit contribution for each model x sold and a $50 profit contribution for each model y sold, what is the optimal sales goal for the company for the next 4-week period?

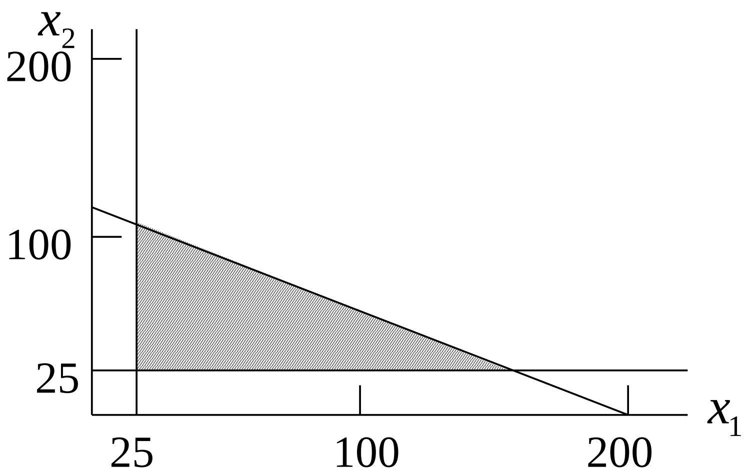
c. Develop a constraint and show the feasible region if management adds the restriction that Car Phones must sell at least as many model y phones as model x phones.

d. What is the new optimal solution if the constraint in part (c) is added to the problem?

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

a. Let x_1 = units of model X

x_2 = units of model Y



$$\text{Max} \quad 40x_1 + 50x_2$$

s.t.

$$3x_1 + 5x_2 \le 600 \quad \text{Sales time}$$

$$1x_1 \ge 25 \quad \text{Model X}$$

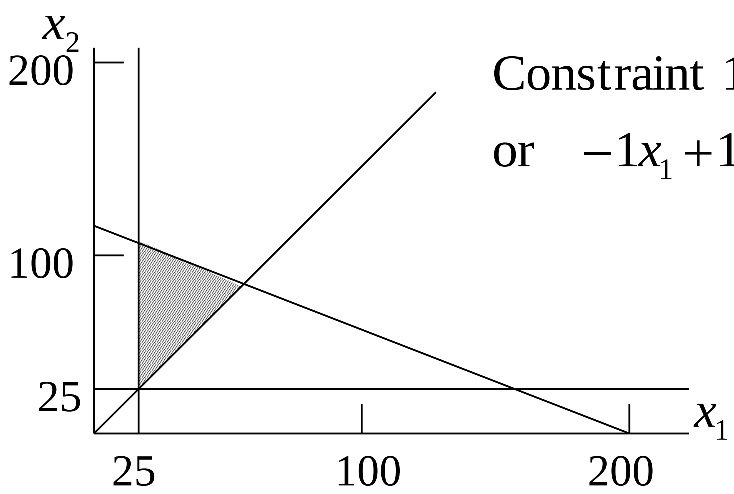
$$1x_2 \ge 25 \quad \text{Model Y}$$

$$x_1, x_2 \ge 0$$

b. Optimal Solution : $x_1 = 158\frac{1}{3}$, $x_2 = 25$, profit $= 7583$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

c.



Constraint $1x_2 \geq 1x_1$

or $-1x_1 + 1x_2 \geq 0$

d. Optimal Solution : $x_1 = 75, x_2 = 75$, profit $= 6750$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Greentree Kennels, Inc., provides overnight lodging for a variety of pets. A particular feature of Greentree is the quality of care the pets receive, including excellent food. The Kennel's dog food is made by mixing two brand-name dog food products to obtain what the kennel calls the "well-balanced dog diet". The data for the two dog foods are as follows:

Dog Food	Cost/ Ounce	Protein(%)	Fat(%)
Bark Bits	$.06	30	15
Canine Chow	$.05	20	30

If Greentree wants to be sure that the dogs receive at least 5 ounces of protein and at least 3 ounces of fat per day, what is the minimum cost mix of the two dog food products?

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Let x_1 = ounces of Bark Bits in feed mix

x_2 = ounces of Canine Chow in feed mix

The problem can be formulated and solved as follows:

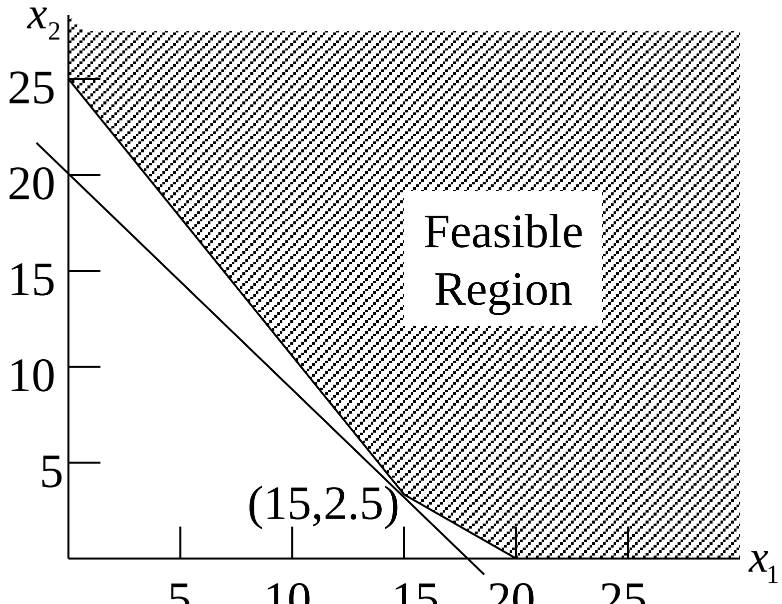
$$\text{Min } .06x_1 + .05x_2$$

s.t.

$$.30x_1 + .20x_2 \geq 5 \quad \text{Protein}$$

$$.15x_1 + .30x_2 \geq 3 \quad \text{Fat}$$

$$x_1, x_2 \geq 0$$



$$Z = .06(15) + .05(5/2) = 1.025$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Bryant's Pizza, Inc., is a producer of frozen pizza products. The company makes a profit of $1.00 for each regular pizza it produces and $1.50 for each deluxe pizza produced. Each pizza includes a combination of dough mix and topping mix. The firm currently has 150 kilos of dough mix and 50 kilos of topping mix. Each regular pizza uses 1 kilo of dough mix and 1/4 kilo of topping mix. Each deluxe pizza uses 1 kilo of sough mix and 1/2 kilo of topping mix. Based on past demand, Bryant can sell at least 50 regular pizzas and at least 25 deluxe pizzas. How many regular and deluxe pizzas should the company make to maximize profits?

a. What is the linear programming model for this problem?

b. Write the linear program in standard form.

c. Find the optimal solution.

d. What are the values and interpretations of all slack and surplus variables?

e. Which constraints are binding?

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

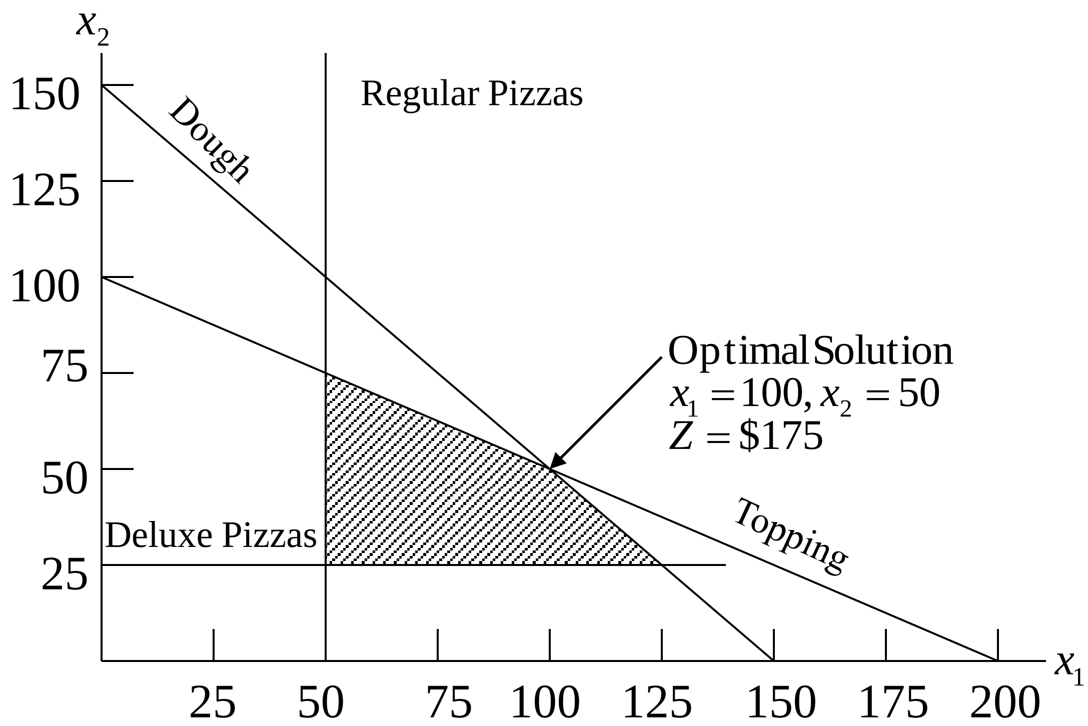
a, b Let $x_1 =$ number of regular pizzas
$x_2 =$ number of deluxe pizzas

$$
\begin{aligned}
&\text{Max } 1x_1 + 1.5x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4 \\
&\text{s.t.} \\
&1x_1 + 1x_2 + 1s_1 = 150 \quad \text{Dough} \\
&\tfrac{1}{4}x_1 + \tfrac{1}{2}x_2 + 1s_2 = 50 \quad \text{Topping} \\
&1x_1 - 1s_3 = 50 \quad \text{Regular} \\
&1x_2 - 1s_4 = 25 \quad \text{Deluxe} \\
&x_1, x_2, s_1, s_2, s_3, s_4 \geq 0
\end{aligned}
$$

$$
\begin{aligned}
&\text{Max } 1x_1 + 1.5x_2 \\
&\text{s.t.} \\
&1x_1 + 1x_2 \leq 150 \quad \text{Dough} \\
&\tfrac{1}{4}x_1 + \tfrac{1}{2}x_2 \leq 50 \quad \text{Topping} \\
&1x_1 \geq 50 \quad \text{Regular} \\
&1x_2 \geq 25 \quad \text{Deluxe} \\
&x_1, x_2 \geq 0
\end{aligned}
$$

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

c.



d. $s_1 = 0$ is the amount of dough not used

$s_2 = 0$ is the amount of topping not used

$s_3 = 50$ is the number of regular pizzas over the minimum demand

$s_4 = 25$ is the number of deluxe pizzas over the minimum demand

e. Binding constraints: dough($s_1 = 0$) and topping($s_2 = 0$) constraints.

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

Consider the linear program given below:

$$
\begin{aligned}
\text{Max} \quad & 5x_1 + 7x_2 \\
\text{s.t.} \quad & \\
& 2x_1 + x_2 \geq 3 \\
& -x_1 + 5x_2 \geq 4 \\
& 2x_1 - 3x_2 \leq 6 \\
& 3x_1 + 2x_2 \leq 35 \\
& {}^{3}\!/_{7}\, x_1 + x_2 \leq 10 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

a. Solve this problem using the graphical solution procedure.

b. Compute the range of optimality for c_1.

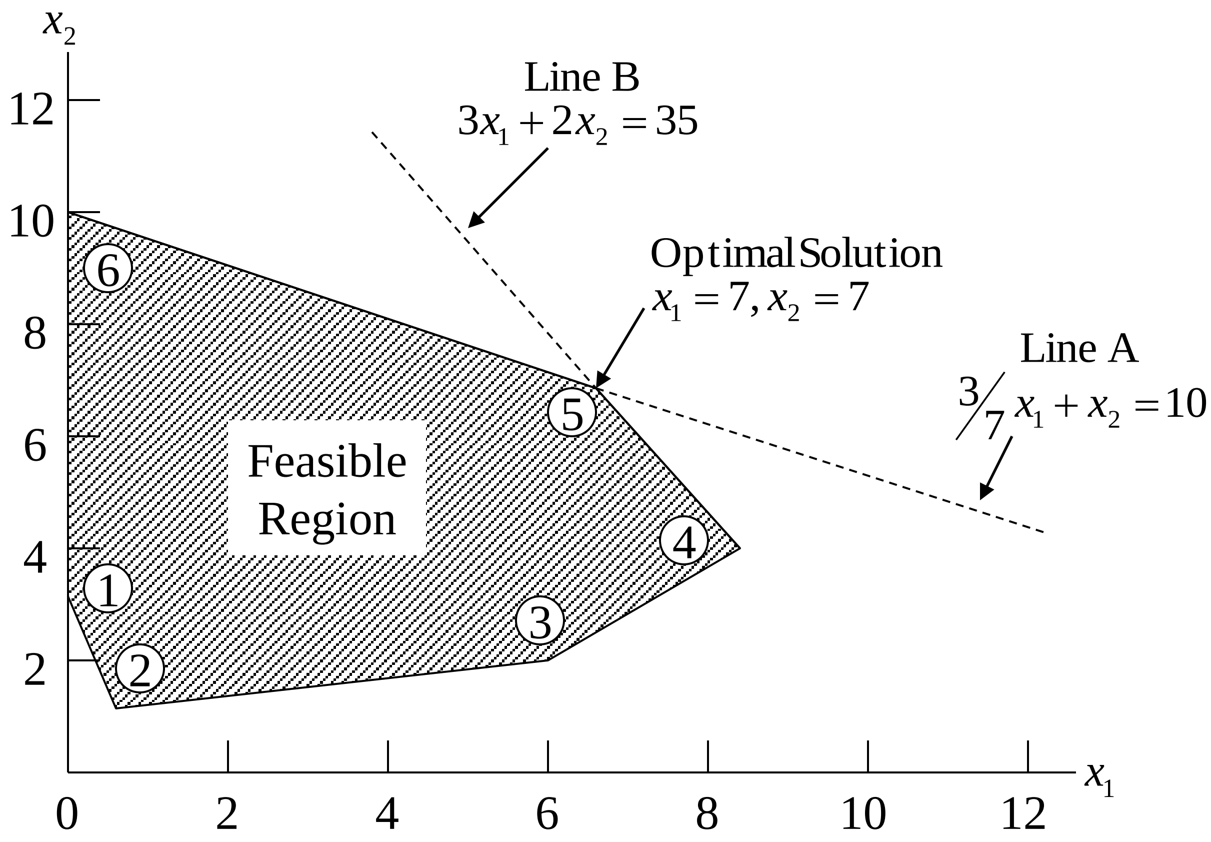
c. Compute the range of optimality for c_2.

d. Suppose c_1 is decreased to 2. What is the new optimal solution?

e. Suppose c_2 is increased to 10. What is the new optimal solution?

ΓΡΑΦΙΚΗ ΕΠΙΛΥΣΗ ΓΠ
GRAPHICAL SOLUTION FOR LPs

a.



b. Slope of Line B $= -3/2$

Slope of Line A $= -3/7$

$$-3/2 \le -c_1/7 \le -3/7$$
$$3/2 \ge c_1/7, \quad c_1/7 \ge -3/7$$
$$c_1 \le 21/2, \quad c_1 \ge 3$$

Range : $3 \le c_1 \le 10.5$

c.
$$-3/2 \le -5/c_2 \le -3/7$$
$$3/2 \ge 5/c_2, \quad 5/c_2 \ge 3/7$$
$$c_2 \ge 10/3, \quad c_2 \le 35/3$$

Range : $10/3 \le c_2 \le 35/3$

d. This change moves c_1 outside its range of optimality. The new optimal solution is found at extreme point 6. It is $x_1 = 0, x_2 = 10$. The value is 70.

e. Since this change leaves c_2 in its range of optimality, the same solution, with value of 105, is optimal.