EXN210: ΜΑΘΗΜΑΤΙΚΕΣ ΜΕΘΟΔΟΙ ΣΤΑ ΟΙΚΟΝΟΜΙΚΑ ΚΑΙ ΔΙΟΙΚΗΣΗ ΙΙ

ΕΠΙΛΥΣΗ ΓΡΑΜΜΙΚΩΝ ΠΡΟΓΡΑΜΜΑΤΩΝ ΜΕ ΗΥ

Σημειώσεις από το ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΥΠΡΟΥ (ΔΙΟΙΚΗΤΙΚΗ ΕΠΙΣΤΗΜΗ)

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Par, Inc., is a small manufacturer of golf equipment and supplies whose management has decided to move into the market for medium- and high-priced golf bags. Par's distributor is enthusiastic about the new product line and has agreed to buy all the golf bags Par produces over the next 3 months.

After a thorough investigation of the steps involved in manufacturing a golf bag, management has determined that each golf bag produced will require the following operations:

1. Cutting and dyeing the material
2. Sewing
3. Finishing (inserting umbrella holder, club separators, etc.)
4. Inspection and packaging.

The director of manufacturing has analyzed each of the operations and concluded that if the company produces a medium-priced standard model, each bag will require 7/10 hour in the cutting and dyeing department, 1/2 hour in the sewing department, 1 hour in the finishing department, and 1/10 hour in the inspection and packaging department. The more expensive deluxe model will require 1 hour for cutting and dyeing, 5/6 hour for sewing, 2/3 hour for finishing, and 1/4 hour for inspection and packaging.

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

This production information is summarized in the table below.

The accounting department has analysed these production figures, assigned all relevant variable costs, and arrived at prices for both bags that will result in a profit contribution[1] of $10 for every standard bag and $9 for every deluxe bag produced.

In addition, after studying departmental work load projections, the director of manufacturing estimates that 630 hours for cutting and dyeing, 600 hours for sewing, 708 hours for finishing, and 135 hours for inspection and packaging will be available for the production of golf bags during the next 3 months.

[1]From an accounting perspective, this is more correctly described as the contribution margin per bag: for example, overhead and other shared costs have not been allocated.

	Production Time (hours)			
Product	Cutting and Dyeing	Sewing	Finishing	Inspection and Packaging
Standard bag	7/10	1/2	1	1/10
Deluxe bag	1	5/6	2/3	1/4

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

$$\max \quad 10x_1 + 9x_2$$

subject to (s.t)

$$7/10\, x_1 + 1x_2 \leq 630 \quad \text{Cutting and dyeing}$$

$$1/2\, x_1 + 5/6\, x_2 \leq 600 \quad \text{Sewing}$$

$$1x_1 + 2/3\, x_2 \leq 708 \quad \text{Finishing}$$

$$1/10\, x_1 + 1/4\, x_2 \leq 135 \quad \text{Inspection and packaging}$$

$$x_1, x_2 \geq 0$$

x_1 = number of standard bags Par, Inc., produces

x_2 = number of deluxe bags Par, Inc., produces

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Objective Function Value = 7667.99463

Variable	Value	Reduced Costs
X1	539.99841	0.00000
X2	252.00113	0.00000

Constraint	Slack/Surplus	Dual Prices
1	0.00000	4.37496
2	120.00070	0.00000
3	0.00000	6.93753
4	17.99988	0.00000

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	6.30000	10.00000	13.49993
X2	6.66670	9.00000	14.28572

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	495.59998	630.00000	682.36316
2	479.99930	600.00000	No Upper Limit
3	580.00146	708.00000	900.00000
4	117.00012	135.00000	No Upper Limit

PLEASE PRESS RETURN TO CONTINUE

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

LINEAR PROGRAMMING PROBLEM

MAX 10X1 +9X2

S.T.

1) .7X1+X2 < 630
2) .5X1+.83333X2 <600
3) 1X1+.66667X2 <708
4) .1X1+.25X2 <135

OPTIMAL SOLUTION

Objective Function Value = 7667.99463

Variable	Value	Reduced Costs
X1	539.99841	0.00000
X2	252.00113	0.00000

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ΜΕΙΩΜΕΝΟ ΚΟΣΤΟΣ
REDUCED COST

- Οι αριθμοί στη στήλη του **ΜΕΙΩΜΕΝΟΥ ΚΟΣΤΟΥΣ** δείχνουν πόσο θα πρέπει να βελτιωθεί κάθε συντελεστής στην αντικειμενική συνάρτηση (για προβλήματα μεγιστοποίησης βελτίωση σημαίνει αύξηση του συντελεστή ενώ για προβλήματα ελαχιστοποίησης βελτίωση σημαίνει μείωση του συντελεστή) για να είναι δυνατόν η μεταβλητή να πάρει θετική τιμή στη βέλτιστη λύση. Εάν μια μεταβλητή έχει θετική τιμή στη βέλτιστη λύση, τότε το μειωμένο κόστος της είναι ίσο με μηδέν.
  - » The information in the REDUCED COST column indicates how much the objective function coefficient of each decision variable would have to improve (for a maximization problem improve means get bigger; for a minimization problem improve means get smaller) before it would be possible for that variable to assume a positive value in the optimal solution. If a decision variable is already positive in the optimal solution, its reduced cost is zero.

ΔΥΙΚΕΣ ΤΙΜΕΣ
DUAL PRICES

- Η ΔΥΙΚΗ ΤΙΜΗ που σχετίζεται με ένα περιορισμό είναι η βελτίωση της τιμής της βέλτιστης λύσης όταν η δεξιά πλευρά του περιορισμού αυξηθεί κατά μια μονάδα.
  – The DUAL PRICE associated with a constraint is the improvement in the value of the solution per-unit increase in the right-hand side of the constraint.

ΔΙΑΣΤΗΜΑΤΑ ΣΥΝΤΕΛΕΣΤΩΝ ΑΝΤΙΚΕΙΜΕΝΙΚΗΣ ΣΥΝΑΡΤΗΣΗΣ
OBJECTIVE COEFFICIENT RANGES

- Τα ΔΙΑΣΤΗΜΑΤΑ ΣΥΝΤΕΛΕΣΤΩΝ ΑΝΤΙΚΕΙΜΕΝΙΚΗΣ ΣΥΝΑΡΤΗΣΗΣ μας λένε ότι εάν οι συντελεστές της αντικειμενικής συνάρτησης παραμείνουν στο διάστημα που υποδεικνύουν, η βέλτιστη λύση που έχει εξευρεθεί θα παραμείνει η βέλτιστη λύση.
  - » The OBJECTIVE FUNCTION COEFFICIENT RANGES tell us that as long the coefficients of the objective function remain in the range indicated, the optimal solution found will remain the optimal solution.

ΔΙΑΣΤΗΜΑΤΑ ΔΕΞΙΑΣ ΠΛΕΥΡΑΣ ΠΕΡΙΟΡΙΣΜΟΥ
RIGHT-HAND SIDE RANGES

- Τα ΔΙΑΣΤΗΜΑΤΑ ΔΕΞΙΑΣ ΠΛΕΥΡΑΣ ΠΕΡΙΟΡΙΣΜΟΥ μας λένε ότι εάν η τιμή της δεξιάς πλευράς του περιορισμού παραμείνει στο διάστημα που υποδεικνύουν, η δυϊκή τιμή του περιορισμού αντιπροσωπεύει τη βελτίωση της τιμής της βέλτιστης λύσης όταν η τιμή της δεξιάς πλευράς του περιορισμού αυξηθεί κατά μια μονάδα.
  - » The RIGHT-HAND SIDE RANGES tell us that as long the right-hand side of a constraint remains in the range indicated, the associated dual price gives the improvement in the value of the optimal solution per-unit increase in the right-hand side.

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ

COMPUTER SOLUTION OF LPs

LINEAR PROGRAMMING PROBLEM

MAX 6.3X1 +9X2

S.T.

1) .7X1+X2 < 630
2) .5X1+.83333X2 <600
3) 1X1+.66667X2 <708
4) .1X1+.25X2 <135

OPTIMAL SOLUTION

Objective Function Value = 5670.00049

Variable	Value	Reduced Costs
X1	539.99841	0.00000
X2	252.00113	0.00000

Constraint	Slack/Surplus	Dual Prices
1	0.00000	9.00000
2	120.00070	0.00000
3	0.00000	0.00000
4	17.99988	0.00000

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	6.30000	6.30000	13.49993
X2	4.20002	9.00000	9.00000

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	495.59998	630.00000	682.36316
2	479.99930	600.00000	No Upper Limit
3	580.00146	708.00000	900.00000
4	117.00012	135.00000	No Upper Limit

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

$$\min \ 2x_1 + 3x_2$$

s.t.

$$1x_1 \geq 125 \quad \text{Demand for product 1}$$
$$1x_1 + 1x_2 \geq 350 \quad \text{Total production}$$
$$2x_2 + 1x_2 \leq 600 \quad \text{Processing time}$$
$$x_1, x_2 \geq 0$$

```
LINEAR PROGRAMMING PROBLEM

MIN 2X1 +3X2

     S.T.

        1)   1X1 > 125
        2)   1X1+1X2 > 350
        3)   2X1+1X2 <600
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ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Objective Function Value = 800.000

Variable	Value	Reduced Costs
X1	250.000	0.000
X2	100.000	0.000

Constraint	Slack/Surplus	Dual Prices
1	125.000	0.000
2	0.000	-4.000
3	0.000	1.000

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	No Lower Limit	2.000	3.000
X2	2.000	3.000	No Upper Limit

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	No Lower Limit	125.000	250.000
2	300.000	350.000	475.000
3	475.000	600.000	700.000

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

The original problem of Par, Inc., is restated below with decimal coefficients:

$$\max \quad 10x_1 + 9x_2$$

subject to (s.t)

$$\begin{aligned} 0.7x_1 + 1x_2 &\le 630 && \text{Cutting and dyeing} \\ 0.5x_1 + 0.83333x_2 &\le 600 && \text{Sewing} \\ 1x_1 + 0.66667x_2 &\le 708 && \text{Finishing} \\ 0.1x_1 + 0.25x_2 &\le 138 && \text{Inspection and packaging} \\ x_1, x_2 &\ge 0 \end{aligned}$$

Recall that x_1 is the number of standard golf bags produced and x_2 is the number of deluxe golf bags produced. Suppose that management is also considering producing a lightweight model designed specifically for golfers who prefer to carry their bags. It is estimated that each lightweight model will require 0.8 hours for cutting and dyeing, 1 hour for sewing, 1 hour for finishing, and 0.25 hours for inspection and packaging. Because of the unique capabilities designed into the new model, Par's management feels that they will realize a profit contribution of $12.85 for each lightweight model produced during the current production period.

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Let us consider the modifications in the original linear programming model that are needed to incorporate the effect of this additional decision variable. We will let x_3 denote the number of lightweight bags produced. After adding x_3 to the objective function and to each of the four constraints, we obtain the following linear program for the modified problem:

$$\begin{aligned}
&\max \quad 10x_1 + 9x_2 + 12.85x_3 \\
&\text{subject to (s.t)} \\
&\qquad 0.7x_1 + 1x_2 + 0.8x_3 \le 630 \quad \text{Cutting and dyeing} \\
&\qquad 0.5x_1 + 0.83333x_2 + 1x_3 \le 600 \quad \text{Sewing} \\
&\qquad 1x_1 + 0.66667x_2 + 1x_3 \le 708 \quad \text{Finishing} \\
&\qquad 0.1x_1 + 0.25x_2 + 0.25x_3 \le 138 \quad \text{Inspection and packaging} \\
&\qquad x_1, x_2, x_3 \ge 0
\end{aligned}$$

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

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LINEAR PROGRAMMING PROBLEM

MAX 10X1 +9X2+12.85X3

     S.T.

        1)   .7X1+X2+.8X3 < 630
        2)   .5X1+.83333X2+X3 <600
        3)   1X1+.66667X2+X3 <708
        4)   .1X1+.25X2+.25X3 <135
```

OPTIMAL SOLUTION

Objective Function Value =   8299.80078

Variable	Value	Reduced Costs
X1	280.00000	0.00000
X2	0.00000	1.15003
X3	428.00000	0.00000

Constraint	Slack/Surplus	Dual Prices
1	91.60001	0.00000
2	32.00000	0.00000
3	0.00000	8.10000
4	0.00000	19.00000

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	5.14000	10.00000	12.07007
X2	No Lower Limit	9.00000	10.15003
X3	11.90907	12.85000	25.00000

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	538.40002	630.00000	No Upper Limit
2	568.00000	600.00000	No Upper Limit
3	540.00000	708.00000	852.63159
4	70.80000	135.00000	114.60001

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Suppose the coefficient of x_2 becomes 10.15003 from its original value of 9

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LINEAR PROGRAMMING PROBLEM

MAX 10X1 +10.15003X2+12.85X3

     S.T.
        1)   .7X1+X2+.8X3 < 630
        2)   .5X1+.83333X2+X3 <600
        3)   1X1+.66667X2+X3 <708
        4)   .1X1+.25X2+.25X3 <135

OPTIMAL SOLUTION

Objective Function Value =        8299.80078
```

Variable	Value	Reduced Costs
X1	403.78317	0.00000
X2	222.81198	0.00000
X3	155.67476	0.00000

Constraint	Slack/Surplus	Dual Prices
1	0.00000	0.00000
2	56.75776	0.00000
3	0.00000	8.10000
4	0.00000	19.00000

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	10.00000	10.00000	12.51072
X2	10.15003	10.15003	15.40790
X3	10.65313	12.85000	12.85000

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	538.40002	630.00000	682.36316
2	543.24225	600.00000	No Upper Limit
3	580.00140	708.00000	852.63159
4	117.00012	135.00000	151.15410

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Suppose we add the constraint $-.3x_1+x_2 \geq 0$

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LINEAR PROGRAMMING PROBLEM

MAX 10X1 + 9X2 + 12.85X3

     S.T.
        1)   .7X1+X2+.8X3 < 630
        2)   .5X1+.83333X2+X3 <600
        3)   1X1+.66667X2+X3 <708
        4)   .1X1+.25X2+.25X3 <135
        5)   -.3X1+X2>0

OPTIMAL SOLUTION
Objective Function Value =        8183.87793
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Variable	Value	Reduced Costs
X1	335.99933	0.00000
X2	100.79980	0.00000
X3	304.80048	0.00000

Constraint	Slack/Surplus	Dual Prices
1	50.16031	0.00000
2	43.20037	0.00000
3	0.00000	7.40998
4	0.00000	21.76006
5	0.00000	-1.38003

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	6.29500	10.00000	12.07007
X2	-3.35000	9.00000	10.15003
X3	11.90907	12.85000	18.14286

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	579.83972	630.00000	No Upper Limit
2	556.79962	600.00000	No Upper Limit
3	540.00000	708.00000	765.00049
4	103.24991	135.00000	147.00008
5	-84.00000	0.00000	101.67704

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

To provide additional practice in formulating and interpreting the computer solution for linear programs involving more than two decision variables, we consider a minimization problem with three decision variables. Bluegrass Farms, located in Lexington Kentucky, has been experimenting with a special diet for its racehorses. The feed components available for the diet are a standard horse feed product, a vitamin-enriched oat product, and a new vitamin and mineral feed additive. The nutritional values in units per pound and the costs for the three feed components are summarized in the table below; for example, each pound of the standard feed component contains 0.8 unit of ingredient A, 1 unit of ingredient B, and 0.1 of ingredient C. The minimum daily diet requirements for each horse are three units of ingredient A, six units of ingredient B, and four units of ingredient C. In addition, to control the weight of the horses, the total daily feed for a horse should not exceed 6 pounds. Bluegrass Farms would like to determine the minimum-cost mix that will satisfy the daily requirements.

Feed Component	**Standard**	**Enriched Oat**	**Additive**
Ingredient A	0.8	0.2	0.0
Ingredient B	1.0	1.5	3.0
Ingredient C	0.1	0.6	2.0
Cost per pound	$0.25	$0.50	$3.00

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

x_1 = number of pounds of the standard horse feed product

x_2 = number of pounds of the enriched oat product

x_3 = number of pounds of the vitamin and mineral feed additive

$$\text{Min} \quad 0.25x_1 + 0.50x_2 + 3x_3$$

s.t.

$$
\begin{aligned}
0.8x_1 + 0.2x_2 \qquad\qquad & \geq 3 \quad \text{Ingredient A} \\
1.0x_1 + 1.5x_2 + 3.0x_3 & \geq 6 \quad \text{Ingredient B} \\
0.1x_1 + 0.6x_2 + 2.0x_3 & \geq 4 \quad \text{Ingredient C} \\
x_1 + \quad x_2 + \quad x_3 & \leq 6 \quad \text{Weight} \\
x_1, x_2, x_3 & \geq 0
\end{aligned}
$$

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

LINEAR PROGRAMMING PROBLEM

MIN .25X1 +.5X2+3X3

S.T.

1) .8X1+.2X2 > 3
2) 1X1+1.5X2+3X3 > 6
3) .1X1+.6X2+2X3 > 4
4) 1X1+1X2+1X3 < 6

OPTIMAL SOLUTION

Objective Function Value = 5.973

Variable	Value	Reduced Costs
X1	3.514	0.000
X2	0.946	0.000
X3	1.541	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	-1.216
2	3.554	0.000
3	0.000	-1.959
4	0.000	0.919

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	-0.393	0.250	No Upper Limit
X2	No Lower Limit	0.500	0.925
X3	1.522	3.000	No Upper Limit

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	1.143	3.000	3.368
2	No Lower Limit	6.000	9.554
3	2.100	4.000	4.875
4	5.562	6.000	8.478

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Electronic Communications manufactures portable radio systems that can be used for two-way communications. The company's new product, which has a range of up to 25 miles, is particularly suitable for use in a variety of business and personal applications. The distribution channels for the new radio are as follows:

1. Marine equipment distributors
2. Business equipment distributors
3. National chain of retail stores
4. Mail order

Because of differing distribution and promotional costs, the profitability of the product will vary with the distribution channel. In addition, the advertising cost and the personal sales effort required will vary with the distribution channels.

The table below summarizes the contribution to profit, advertising costs and personal sales effort data pertaining to the Electronic Communications problem. The firm has set the advertising budget at $5000 and there is a maximum of 1800 hours of sales force time available for allocation to the sales effort. Management has also decided to produce exactly 600 units for the current production period.

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

Finally, an ongoing contract with the national chain of retail stores requires that at least 150 units be distributed through this distribution channel.

Electronic Communications is now faced with the problem of establishing a strategy that will provide for the distribution of the radios in such a way that overall profitability of the new radio production will be maximized. Decisions must be made as to how many units should be allocated to each of the four distribution channels, as well as how to allocate the advertising budget and sales force effort to each of the four distribution channels.

Distribution Channel	Profit per Unit Sold	Advertising Cost per Unit Sold	Personal Sales Effort per Unit Sold
Marine distributors	$90	$10	2 hours
Business distributors	$84	$8	3 hours
National Retail Stores	$70	$9	3 hours
Mail order	$60	$15	None

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

x_1 = number of units produced for the marine equipment distribution channel

x_2 = number of units produced for the business equipment distribution channel

x_3 = number of units produced for the national retail chain distribution channel

x_3 = number of units produced for the mail - order distribution channel

$$\text{Max} \quad 90x_1 + 84x_2 + 70x_3 + 60x_4$$

s.t.

$$\begin{aligned}
10x_1 + 8x_2 + 9x_3 + 15x_4 &\le 5000 \quad \text{Advertising budget} \\
2x_1 + 3x_2 + 3x_3 &\le 1800 \quad \text{Sales force availability} \\
1x_1 + 1x_2 + 1x_3 + 1x_4 &= 600 \quad \text{Production level} \\
x_3 &\ge 150 \quad \text{Retail stores requirements} \\
x_1, x_2, x_3, x_4 &\ge 0
\end{aligned}$$

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

LINEAR PROGRAMMING PROBLEM

MAX 90X1 +84X2+70X3+60X4

S.T.

1) 10X1+8X2+9X3+15X4 < 5000
2) 2X1+3X2+3X3 < 1800
3) 1X1+1X2+1X3+1X4 = 600
4) X3 > 150

OPTIMAL SOLUTION
Objective Function Value = 48450.000

Variable	Value	Reduced Costs
X1	25.000	0.000
X2	425.000	0.000
X3	150.000	0.000
X4	0.000	45.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	3.000
2	25.000	0.000
3	0.000	60.000
4	0.000	-17.000

ΕΠΙΛΥΣΗ ΓΠ ΜΕ ΗΥ
COMPUTER SOLUTION OF LPs

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	84.000	90.000	No Upper Limit
X2	50.000	84.000	90.000
X3	No Lower Limit	70.000	87.000
X4	No Lower Limit	60.000	105.000

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	4950.000	5000.000	5850.000
2	1775.000	1800.000	No Upper Limit
3	515.000	600.000	603.571
4	0.000	150.000	200.000

ΠΑΡΑΛΛΗΛΕΣ ΑΛΛΑΓΕΣ
SIMULTANEOUS CHANGES

- **Ο 100% Κανόνας για Συντελεστές της Αντικειμενικής Συνάρτησης**
  - Για όλους τους συντελεστές της αντικειμενικής συνάρτησης που αλλάζουν, προσθέστε τις επι τοις εκατό επιτρεπτές μειώσεις ή αυξήσεις που προκαλούνται από τις αλλαγές. Εάν το άθροισμα δεν υπερβαίνει το 100%, τότε η βέλτιστη λύση δεν θα αλλάξει.
    - » 100 PERCENT RULE FOR OBJECTIVE FUNCTION COEFFICIENTS: For all objective function coefficients that are changed, sum the percentages of the allowable increases and the allowable decreases represented by the changes. If the sum of the percentage changes does not exceed 100%, the optimal solution will not change.

ΠΑΡΑΛΛΗΛΕΣ ΑΛΛΑΓΕΣ: ΠΑΡΑΔΕΙΓΜΑ
SIMULTANEOUS CHANGES: EXAMPLE

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LINEAR PROGRAMMING PROBLEM

MAX 10X1 +9X2

     S.T.

        1)   .7X1+X2 < 630
        2)   .5X1+.83333X2 <600
        3)   1X1+.66667X2 <708
        4)   .1X1+.25X2 <135

OPTIMAL SOLUTION
Objective Function Value =        7667.99463
```

Variable	Value	Reduced Costs
X1	539.99841	0.00000
X2	252.00113	0.00000

Constraint	Slack/Surplus	Dual Prices
1	0.00000	4.37496
2	120.00070	0.00000
3	0.00000	6.93753
4	17.99988	0.00000

ΠΑΡΑΛΛΗΛΕΣ ΑΛΛΑΓΕΣ: ΠΑΡΑΔΕΙΓΜΑ
SIMULTANEOUS CHANGES: EXAMPLE

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	6.30000	10.00000	13.49993
X2	6.66670	9.00000	14.28572

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	495.59998	630.00000	682.36316
2	479.99930	600.00000	No Upper Limit
3	580.00146	708.00000	900.00000
4	117.00012	135.00000	No Upper Limit

ΠΑΡΑΛΛΗΛΕΣ ΑΛΛΑΓΕΣ:ΠΑΡΑΔΕΙΓΜΑ
SIMULTANEOUS CHANGES:EXAMPLE

$$\text{Maximum Allowable Increase}: 13.49993 - 10 = 3.49993 \text{ (for } c_1)$$

$$\text{Suppose the coefficient } c_1 = 10 \text{ becomes } 11.50$$

$$(100)\frac{1.50}{3.49993} = 42.81\% \text{ of the allowable increase}$$

$$\text{Maximum Allowable Decrease}: 9 - 6.6667 = 2.3333 \text{ (for } c_2)$$

$$\text{Suppose the coefficient } c_2 = 9 \text{ becomes } 8.25$$

$$(100)\frac{0.75}{2.3333} = 32.14\% \text{ of the allowable decrease}$$

$$42.86\% + 32.14\% = 75\% \leq 100\%$$

The above changes will not affect the optimal solution

ΠΑΡΑΛΛΗΛΕΣ ΑΛΛΑΓΕΣ
SIMULTANEOUS CHANGES

- **Ο 100% Κανόνας για τις Τιμές στη Δεξιά Πλευρά Περιορισμού**
  - Για όλες τις δεξιές πλευρές περιορισμών που αλλάζουν, προσθέστε τις επι τοις εκατό επιτρεπτές μειώσεις ή αυξήσεις που προκαλούνται από τις αλλαγές. Εάν το άθροισμα δεν υπερβαίνει το 100%, τότε η δυϊκές τιμές δεν θα αλλάξουν.
    - » 100 PERCENT RULE FOR CONSTRAINT RIGHT-HAND SIDES: For all right-hand sides that are changed, sum the percentages of the allowable increases and decreases. If the sum of the percentages does not exceed 100%, then the dual prices will not change.

ΠΑΡΑΛΛΗΛΕΣ ΑΛΛΑΓΕΣ: ΠΑΡΑΔΕΙΓΜΑ
SIMULTANEOUS CHANGES: EXAMPLE

$$\text{Allowable Increases} : 682.36316 - 630 = 52.36316$$

$$: 900 - 708 \qquad = 192$$

Suppose 630 becomes 650

$$(100)\frac{20}{52.36316} = 38.19\%$$

Suppose 708 becomes 808

$$(100)\frac{100}{192} = 52.08\%$$

$$38.19\% + 52.08\% = 90.27\%$$

The above changes will not change the dual prices