EXN210: ΜΑΘΗΜΑΤΙΚΕΣ ΜΕΘΟΔΟΙ ΣΤΑ ΟΙΚΟΝΟΜΙΚΑ ΚΑΙ ΔΙΟΙΚΗΣΗ ΙΙ

ΕΦΑΡΜΟΓΕΣ ΓΡΑΜΜΙΚΩΝ ΠΡΟΓΡΑΜΜΑΤΩΝ

Σημειώσεις από to ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΥΠΡΟΥ (ΔΙΟΙΚΗΤΙΚΗ ΕΠΙΣΤΗΜΗ)

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΟ ΜΑΡΚΕΤΙΝΓΚ
LP MARKETING APPLICATIONS

Relax-and-Enjoy Lake Development Corporation is developing a lakeside community at a privately owned lake. The primary market for the lakeside lots and homes they hope to sell includes all middle- and upper-income families within approximately 100 miles of the development. Relax-and-Enjoy has employed the advertising firm of Boone, Philips and Jackson (BP&J) to design the promotional campaign for the project.

After considering possible advertising media and the market to be covered, BP&J has recommended that the first month's advertising be restricted to five sources. At the end of the month, BP&J will then reevaluate its strategy based on the month's results. BP&J has collected data on the number of potential purchase families reached, the cost per advertisement, the maximum number of times each is available, and the media exposure rating for each of the five media. The exposure rating is measured in terms of an exposure unit, a measure of the relative value of one advertisement in each of the media. These measures, based on BP&J's experience in the advertising business, take into account such factors as audience profile (age, income, and education of the audience reached), image presented, and quality of the advertisement. The information collected is presented in the Table that follows. Relax-and-Enjoy has provided BP&J with an advertising budget of $30,000 for the first's month campaign.

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In addition, Relax-and-Enjoy has imposed the following restrictions on how BP&J may allocate these funds: at least 10 television commercials must be used, at least 50,000 potential purchases must be reached during the month, and no more than $18,000 may be spent on television advertisements. What advertising media selection plan should the advertising firm recommend?

Advertising Media	**Number of Potential Purchase Families Reached**	**Cost Per Advertisement**	**Maximum Times Available Per Month***	**Exposure Units**
1. Daytime TV (1 min) station WKLA	1000	$1500	15	65
2. Evening TV (30 sec), station WKLA	2000	$3000	10	90
3. Daily newspaper (full page), *The Morning Journal*	1500	$ 400	25	40
4. Sunday newspaper magazine (1/2 page color). *The Sunday Press*	2500	$1000	4	60
5. Radio, 8:00 A.M. or 5:00 P.M. news (30 sec), station KNOP	300	$ 100	30	20

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We begin by defining the decisions variables as follows:

x_1 = number of times daytime TV is used
x_2 = number of times evening TV is used
x_3 = number of times daily newspaper is used
x_4 = number of times Sunday newspaper is used
x_5 = number of times radio is used

$$\text{Max} \quad 65x_1 + 90x_2 + 40x_3 + 60x_4 + 20x_5$$

The constraints for the model can be formulated from the information given

x_1	≤ 15	
x_2	≤ 10	Availability
x_3	≤ 25	of
x_4	≤ 4	media
x_5	≤ 30	
$1500x_1 + 3000x_2 + 400x_3 + 1000x_4 + 100x_5$	$\leq 30{,}000$	Budget
$x_1 + x_2$	≥ 10	Television
$1500x_1 + 3000x_2$	$\leq 18{,}000$	restrictions
$1000x_1 + 2000x_2 + 1500x_3 + 2500x_4 + 300x_5$	$\geq 50{,}000$	Families reached

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΟ ΜΑΡΚΕΤΙΝΓΚ
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Media	Frequency	Budget
Daytime TV	10	$15,000
Daily newspaper	25	10,000
Sunday newspaper	2	2,000
Radio	30	3,000
		$30,000

Total audience contacted = 61,500
Exposure units = 2,370

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Market Survey, Inc. (MSI), specializes in evaluating consumer reaction to new products, services, and advertising campaigns. A client firm has requested assistance from MSI in ascertaining consumer reaction to a recently marketed product for household use. During meetings with the client it was agreed that door-to-door personal interviews would be used to obtain information from both households with children and households without children. In addition, it was agreed that both day and evening interviews would be conducted to allow for a variety of household work schedules. Specifically, the client's contract called for MSI to conduct 1000 interviews with the following quota guidelines:

1. At least 400 households with children would be interviewed.
2. At least 400 households without children would be interviewed.
3. The total number of households interviewed during the evening would be at least as great as the number of households interviewed during the day.
4. At least 40% of the interviews for households with children would be conducted during the evening.
5. At least 60% of the interviews for households without children would be conducted during the evening.

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Since the interviews for households with children take additional interviewer time, and since evening interviewers are paid more than daytime interviewers, the cost of an interview varies with the type of the interview. Based on previous research studies, estimates of the interview costs are as follows:

	Interview Cost	
Household	**Day**	**Evening**
Children	$20	$25
No children	$18	$20

What is the household, time-of-day interview plan that will satisfy the contract requirements at a minimum total interviewing cost?

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x_{11} = the number of interviews for households with children that are conducted during the day

x_{12} = the number of interviews for households with children that are conducted during the evening

x_{21} = the number of interviews for households without children that are conducted during the day

x_{22} = the number of interviews for households without children that are conducted during the evening

$$\text{Min } 20x_{11} + 25x_{12} + 18x_{21} + 20x_{22}$$

The constraint requiring a total of 1000 interviews is written as

$$x_{11} + x_{12} + x_{21} + x_{22} = 1000$$

The five specifications concerning the types of interviews are as follows.

- Households with children :

$$x_{11} + x_{12} \geq 400$$

- Households without children :

$$x_{21} + x_{22} \geq 400$$

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- At least as many evening interviews as day interviews:

$$x_{12} + x_{22} \geq x_{11} + x_{21}$$

The usual format for linear programming model formulation and computer input places all decision variables on the left-hand side of the inequality and a constant (possibly zero) on the right-hand side. Thus, we will rewrite this constraint as

$$-x_{11} + x_{12} - x_{21} + x_{22} \geq 0$$

- At least 40% of interviews for households with children during the evening:

$$x_{12} \geq 0.4(x_{11} + x_{12})$$

or

$$-0.4x_{11} + 0.6x_{12} \geq 0$$

- At least 60% of interviews for households without children during the evening:

$$x_{22} \geq 0.6(x_{21} + x_{22})$$

or

$$-0.6x_{21} + 0.4x_{22} \geq 0$$

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$$\text{Min } 20x_{11} + 25x_{12} + 18x_{21} + 20x_{22}$$

s.t.

$$\begin{array}{llll}
x_{11} + x_{12} + x_{21} + x_{22} = 1000 & \text{Total interviews} \\
x_{11} + x_{12} \geq 400 & \text{Households with children} \\
x_{21} + x_{22} \geq 400 & \text{Households without children} \\
-x_{11} + x_{12} - x_{21} + x_{22} \geq 0 & \text{Evening interviews} \\
-0.4x_{11} + 0.6x_{12} \geq 0 & \text{Evening households with children} \\
-0.6x_{21} + 0.4x_{22} \geq 0 & \text{Evening households without children} \\
x_{11}, x_{12}, x_{21}, x_{22} \geq 0 &
\end{array}$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΟ ΜΑΡΚΕΤΙΝΓΚ
LP MARKETING APPLICATIONS

Objective Function Value = 20320.000

Variable	Value	Reduced Costs
X11	240.000	0.000
X12	160.000	0.000
X21	240.000	0.000
X22	360.000	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	-19.200
2	0.000	-2.800
3	200.000	0.000
4	40.000	0.000
5	0.000	-5.000
6	0.000	-2.000

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
LP FINANCE APPLICATIONS

Consider the case Welte Mutual Funds, Inc., located in New York City. Welte has just obtained $100,000 by converting industrial bonds to cash and is now looking for other investment opportunities for these funds. Considering Welte's current investments, the firm's top financial analyst recommends that all new investments should be made in the oil or steel industry, or in government bonds. Specifically, the analyst has identified five investment opportunities and projected their annual rates of return. The investments and rates of return are shown in the Table that follows.

Management of Welte has imposed the following investment guidelines:

1. Neither industry (oil or steel) should receive more than $50,000 of the total investment.
2. Government bonds should be at least 25% of the steel industry investments.
3. The investment in Pacific Oil, the high-return but high-risk investment, cannot be more than 60% of the total oil industry investment.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
LP FINANCE APPLICATIONS

What portfolio recommendations-investments and amounts-should be made for the available $100,000? Given the objective of maximizing projected return subject to the budgetary and managerially imposed constraints, we can answer this question by formulating a linear programming model of the problem. The solution to this linear programming model will then provide investment recommendations for the management of Welte Mutual Funds.

Investment	**Projected Rate of Return (%)**
Atlantic Oil	7.3
Pacific Oil	10.3
Midwest Steel	6.4
Huber Steel	7.5
Government bonds	4.5

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
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Let

x_1 = dollars invested in Atlantic Oil
x_2 = dollars invested in Pacific Oil
x_3 = dollars invested in Midwest Steel
x_4 = dollars invested in Huber Steel
x_5 = dollars invested in Government bonds

Using the projected rates of return shown in the Table, the objective function for maximizing the total return for the portfolio can be written as

$$\text{Max } 0.073x_1 + 0.103x_2 + 0.064x_3 + 0.075x_4 + 0.045x_5$$

The constraint specifying the available \$100,000 is written as

$$x_1 + x_2 + x_3 + x_4 + x_5 = 100{,}000$$

The requirements that neither the oil nor the steel industry should receive more than \$50,000 are as follows:

$$x_1 + x_2 \le 50{,}000$$
$$x_3 + x_4 \le 50{,}000$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
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The requirement that government bonds be at least 25% of the steel industry investment is expressed as follows:

$$x_5 \geq 0.25(x_3 + x_4)$$

or

$$-0.25x_3 - 0.25x_4 + x_5 \geq 0$$

Finally, the constraint that Pacific Oil cannot be more than 60% of the total oil industry investment becomes

$$x_2 \leq 0.60(x_1 + x_2)$$

or

$$-0.60x_1 + 0.40x_2 \leq 0$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
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Max $0.073x_1 + 0.103x_2 + 0.064x_3 + 0.075x_4 + 0.045x_5$

s.t.

$x_1 + x_2 + x_3 + x_4 + x_5 = 100{,}000$		Available funds
$x_1 + x_2 \le 50{,}000$		Oil industry maximum
$x_3 + x_4 \le 50{,}000$		Steel industry maximum
$-0.25x_3 - 0.25x_4 + x_5 \ge 0$		Government bonds minimum
$-0.6x_1 + 0.4x_2 \le 0$		Pacific Oil restriction

$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
LP FINANCE APPLICATIONS

Objective Function Value = 8000.000

Variable	Value	Reduced Costs
X1	20000.000	0.000
X2	30000.000	0.000
X3	0.000	0.011
X4	40000.000	0.000
X5	10000.000	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	0.069
2	0.000	0.022
3	10000.000	0.000
4	0.000	-0.024
5	0.000	0.030

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
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Investment	**Amount**	**Expected Annual Return**
Atlantic Oil	$ 20,000	$1,460
Pacific Oil	30,000	3,090
Huber Steel	40,000	3,000
Government bonds	10,000	450
	$100,000	$8,000

Expected annual return of $8,000

Overall rate of return= 8%

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

McCormick Manufacturing Company produces two products with contributions to per-unit profit of $10 and $9, respectively. The labor requirements per unit produced and the total hours available from personnel assigned to each of four departments are shown in the Table below.

	Labor-Hours Per Unit		
Department	**Product 1**	**Product 2**	**Total Hours Available**
1	0.65	0.95	6500
2	0.45	0.85	6000
3	1.00	0.70	7000
4	0.15	0.30	1400

Assuming that the number of hours available in each department is fixed, we can formulate McCormick's problem as a standard product-mix linear program with the following decision variables:

$$x_1 = \text{units of product 1}$$
$$x_2 = \text{units of product 2}$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

The linear program can be written as

$$\begin{aligned} &\text{Max } 10x_1 + 9x_2 \\ &\text{s.t.} \\ &\quad 0.65x_1 + 0.95x_2 \le 6500 \\ &\quad 0.45x_1 + 0.85x_2 \le 6000 \\ &\quad 1.00x_1 + 0.70x_2 \le 7000 \\ &\quad 0.15x_1 + 0.30x_2 \le 1400 \\ &\quad x_1, x_2 \ge 0 \end{aligned}$$

The optimal solution to the linear programming model provides 5744 units of product 1, 1795 units of product 2, and a total profit of $73,590. With this optimal solution, departments 3 and 4 are operating at capacity while departments 1 and 2 have a slack of approximately 1062 and 1890 hours, respectively. We would anticipate that the product mix would change and that the total profit would increase if the work force assignment could be revised so that the slack, or unused hours in departments 1 and 2 could be transferred to the departments currently working at capacity.

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However, the production manager may be uncertain as to how the work force should be reallocated among the four departments. Let us expand the linear programming model to include decision variables that will help determine the optimal work force allocation in addition to the profit maximizing product mix.

Suppose McCormick has a cross-training program that enables some employees to be transferred between departments. By taking advantage of the cross-training skills, a limited number of employees and labor-hours may be transferred from one department to another. For example, suppose that the cross-training permits transfers as shown in the Table that follows. Row 1 of this table shows that some employees assigned to department 1 have cross-training skills that permit them to be transferred to departments 2 and 3. The column in the right-hand margin of the Table shows that for the current production planning period, a maximum of 400 hours can be transferred out of department 1. Similar cross-training capabilities and capacities are shown for departments 2,3 and 4.

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	Cross-Training Transfers Permitted to Department				
From Department	**1**	**2**	**3**	**4**	**Maximum Hours Transferable**
1	-	yes	yes	-	400
2	-	-	yes	yes	800
3	-	-	-	yes	100
4	yes	yes	-	-	200

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
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When work force assignments are flexible, we do not automatically know how many hours of labor should be assigned to or transferred from each department. Let us add the following decision variables to the linear programming model:

$$b_i = \text{the labor-hours allocated to department } i \text{ for } i = 1,2,3, \text{ and } 4$$

$$t_{ij} = \text{the labor-hours transferred from department } i \text{ to department } j$$

With the addition of decision variables b_1, b_2, b_3, and b_4 the capacity restrictions for the four departments can be written as follows:

$$0.65x_1 + 0.95x_2 \le b_1$$
$$0.45x_1 + 0.85x_2 \le b_2$$
$$1.00x_1 + 0.70x_2 \le b_3$$
$$0.15x_1 + 0.30x_2 \le b_4$$

Since b_1, b_2, b_3, and b_4 are now decision variables, we follow the standard practice of placing these variables on the left-hand side of the inequalities.

Therefore, the first four constraints of the linear programming model become

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
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$$
\begin{aligned}
0.65x_1 + 0.95x_2 - b_1 &\le 0 \\
0.45x_1 + 0.85x_2 - b_2 &\le 0 \\
1.00x_1 + 0.70x_2 - b_3 &\le 0 \\
0.15x_1 + 0.30x_2 - b_4 &\le 0
\end{aligned}
$$

The labor-hours ultimately allocated to each department must be determined by a series of balance equations or constraints that include the number of hours initially assigned to each department plus the number of hours transferred into the department minus the number of hours transferred out of the department. Using department 1 as an example, the work force allocation is determined as follows:

$$b_1 = \begin{pmatrix} \text{Hours initially} \\ \text{in department 1} \end{pmatrix} + \begin{pmatrix} \text{Hours transferred} \\ \text{into department 1} \end{pmatrix} - \begin{pmatrix} \text{Hours transferred} \\ \text{out of department 1} \end{pmatrix}$$

We know that 6500 hours were initially assigned to department 1. We use the transfer decision variables t_{i1} to denote transfers into department 1 and t_{1i} to denote transfers out of department 1. The cross-training capabilities involving department 1 are restricted to transfers from department 4 (variable t_{41}) and transfers out to either department 2 or department 3 (variables t_{12} and t_{13}). Thus, we can express the total work force allocation for department 1 as follows:

$$b_1 = 6500 + t_{41} - t_{12} - t_{13}$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Moving the decision variables for the work force transfers to the left-hand side, we have the balance equation or constraint

$$b_1 - t_{41} + t_{12} + t_{13} = 6500$$

A constraint of the above form will be needed for each of the four departments. Thus the following balance constraints for departments 2,3, and 4 would be added to the model.

$$b_2 - t_{12} - t_{42} + t_{23} + t_{24} = 6000$$
$$b_3 - t_{13} - t_{23} + t_{34} = 7000$$
$$b_4 - t_{24} - t_{34} + t_{41} + t_{42} = 1400$$

Finally, since the number of hours transferred out of each department is limited, a transfer capacity constraint must be added for each of the four departments. The four additional constraints necessary are:

$$t_{12} + t_{13} \leq 400$$
$$t_{23} + t_{24} \leq 800$$
$$t_{34} \leq 100$$
$$t_{41} + t_{42} \leq 200$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

The complete linear model has two product decision variables (x_1 and x_2), four department work force assignment variables ($b_1, b_2, b_3,$ and b_4), and seven transfer variables ($t_{12}, t_{13}, t_{23}, t_{24}, t_{34}, t_{41}$ and t_{42}). There are a total of 12 constraints.

McCormick's profit can be increased to $84,011 by taking advantage of work force transfers. The optimal product mix of 6825 units of product 1 and 1751 units of product 2 can be achieved if $t_{13} = 400$ hours are transferred from department 1 to department 3; $t_{23} = 751$ hours are transferred from department 2 to department 3; $t_{24} = 49$ hours are transferred from department 2 to department 4; and $t_{34} = 100$ hous are transferred from department 3 to department 4. The resulting work force assignments for departments 1-4 would provide 6100, 5200, 8051, and 1549 hours, respectively.

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Objective Function Value = 84011.297

Variable	Value	Reduced Costs
X1	6824.858	0.000
X2	1751.413	0.000
B1	6100.000	0.000
B2	5200.000	0.000
B3	8050.847	0.000
B4	1549.153	0.000
T41	0.000	7.458
T12	0.000	8.249
T13	400.000	0.000
T42	0.000	8.249
T23	750.847	0.000
T24	49.153	0.000
T34	100.000	0.000

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Constraint	Slack/Surplus	Dual Prices
1	0.000	0.791
2	640.113	0.000
3	0.000	8.249
4	0.000	8.249
5	0.000	0.791
6	0.000	0.000
7	0.000	8.249
8	0.000	8.249
9	0.000	7.458
10	0.000	8.249
11	0.000	0.000
12	200.000	0.000

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΟ ΜΑΡΚΕΤΙΝΓΚ
LP MARKETING APPLICATIONS

Media selection. The Westchester Chamber of Commerce periodically sponsors public service seminars and programs. Currently, promotional plans are under way for this year's program. Advertising alternatives include television, radio, and newspaper. Audience estimates, costs, and maximum media usage limitations are shown below.

Constraint	**Television**	**Radio**	**Newspaper**
Audience per advertisement	100,000	18,000	40,000
Cost per advertisement	$2,000	$300	$600
Maximum media usage	10	20	10

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LP MARKETING APPLICATIONS

To ensure a balanced usage of advertising media, radio advertisements must not exceed 50% of the total number of advertisements authorized. In addition, it has been requested that television account for at least 10% of the total number of advertisements authorized.

a. If the promotional budget is limited to \$18,200 how many commercial messages should be run on each medium to maximize total audience contact? What is the allocation of the budget among the three media, and what is the total audience reached?

b. What is the estimated audience contact that would result from an extra \$100 allocated to the advertising budget?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΟ ΜΑΡΚΕΤΙΝΓΚ
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a. Let x_1 = number of television spot advertisements

x_2 = number of radio advertisements

x_3 = number of newspaper advertisements

$$\text{Max} \quad 100{,}000x_1 + 18{,}000x_2 + 40{,}000x_3$$

s.t.

$$\begin{array}{llll}
2{,}000x_1 + 300x_2 + 600x_3 \le 18{,}200 & & \text{Budget} \\
x_1 \le 10 & & \text{Max TV} \\
x_2 \le 20 & & \text{Max Radio} \\
x_3 \le 10 & & \text{Max News} \\
-0.5x_1 + 0.5x_2 - 0.5x_3 \le 0 & & \text{Max 50\% Radio} \\
0.9x_1 - 0.1x_2 - 0.1x_3 \ge 0 & & \text{Min 10\% TV} \\
x_1, x_2, x_3 \ge 0 & &
\end{array}$$

Solution:		Budget $
	$x_1 = 4$	$8,000
	$x_2 = 14$	4,200
	$x_3 = 10$	6,000
		$18,200

Audience = 1,052,000.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΟ ΜΑΡΚΕΤΙΝΓΚ
LP MARKETING APPLICATIONS

Objective Function Value = 1052000.000

Variable	Value	Reduced Costs
X1	4.000	0.000
X2	14.000	0.000
X3	10.000	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	51.304
2	6.000	0.000
3	6.000	0.000
4	0.000	11826.087
5	0.000	5217.392
6	1.200	0.000

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΟ ΜΑΡΚΕΤΙΝΓΚ
LP MARKETING APPLICATIONS

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	-18000.008	100000.000	120000.000
X2	15000.000	18000.000	No Upper Limit
X3	28173.914	40000.000	No Upper Limit

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	14750.000	18200.000	31999.996
2	4.000	10.000	No Upper Limit
3	14.000	20.000	No Upper Limit
4	0.000	10.000	12.339
5	-8.050	0.000	2.936
6	No Lower Limit	0.000	1.200

b. The dual price for the budget constraint is 51.30. Thus, a $100 increase in budget should provide an increase in audience coverage of approximately 5,130. The range of feasibility for the right-hand side of the budget constraint shows that this interpretation is correct.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
LP FINANCE APPLICATIONS

Portfolio selection. National Insurance Associates carries an investment portfolio of a variety of stocks, bonds, and other investment alternatives. Currently $200,000 of funds are available and must be considered for new investment opportunities. The four stock options National is considering and the relevant financial data are as follows:

	Stock			
Financial Data	**A**	**B**	**C**	**D**
Price per share	$100	$50	$80	$40
Annual rate of return	0.12	0.08	0.06	0.10
Risk measure per dollar invested	0.10	0.07	0.05	0.08

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
LP FINANCE APPLICATIONS

The risk measure indicates the relative uncertainty associated with the stock in terms of its realizing the projected annual return; higher values indicate higher risk. The risk measures are provided by the firm´s top financial advisor.

National´s top management has stipulated the following investment guidelines:

1. The annual rate of return for the portfolio must be at least 9%.
2. No one stock can account for more than 50% of the total dollar investment.

a. Use linear programming to develop an investment portfolio that minimizes risk.

b. If the firm ignores risk and uses a maximum return-on-investment strategy, what is the investment portfolio?

c. What is the dollar difference between the portfolios recommended in parts (a) and (b)? Why might the company prefer the model developed in part (a)?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
LP FINANCE APPLICATIONS

Let x_1 = number of shares of stock A

x_2 = number of shares of stock B

x_3 = number of shares of stock C

x_4 = number of shares of stock D

a. To get data on a per share basis multiply the price by the rate of return or the risk measure value

$$\text{Min} \quad 10x_1 + 3.5x_2 + 4x_3 + 3.2x_4$$

s.t.

$$\begin{aligned}
100x_1 + 50x_2 + 80x_3 + 40x_4 &= 200{,}000 \\
12x_1 + 4x_2 + 4.8x_3 + 4x_4 &\geq 18{,}000 \quad (9\% \text{ of } 200{,}000) \\
100x_1 &\leq 100{,}000 \\
50x_2 &\leq 100{,}000 \\
80x_3 &\leq 100{,}000 \\
40x_4 &\leq 100{,}000 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ
LP FINANCE APPLICATIONS

Solution : $x_1 = 333.3, x_2 = 0, x_3 = 833.3, x_4 = 2500$

Risk : 14,666.7

Return : 18,000 (9%) from constraint 2

b.

$$
\begin{aligned}
\text{Max} \quad & 12x_1 + 4x_2 + 4.8x_3 + 4x_4 \\
\text{s.t.} \quad & \\
& 100x_1 + 50x_2 + 80x_3 + 40x_4 = 200{,}000 \\
& 100x_1 \leq 100{,}000 \\
& 50x_2 \leq 100{,}000 \\
& 80x_3 \leq 100{,}000 \\
& 40x_4 \leq 100{,}000 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

Solution : $x_1 = 1000, x_2 = 0, x_3 = 0, x_4 = 2500$

Risk : $10x_1 + 3.5x_2 + 4x_3 + 3.2x_4 = 18{,}000$

Return : 22,000 (11%)

c. The return in part (b) is $4,000 or 2% greater, but the risk index has increased by 3,333.

Obtaining a reasonable return with a lower risk is a preferred strategy in many financial firms. The more speculative, higher return investments are not always preferred because of their associated higher risk.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Purchasing. Edwards Manufacturing Company purchases two component parts from three different suppliers. The suppliers have limited capacity, and no one supplier can meet all of the company's needs. In addition, the suppliers differ in the prices charged for the components. Component price data (in price/unit) are as follows:

	Supplier		
Component	**1**	**2**	**3**
1	$12	$13	$14
2	$10	$11	$10

Each supplier has a limited capacity in terms of the total number of components it can supply. However, as long as Edwards provides sufficient advance orders, each supplier can devote its capacity to component 1, component 2, or any combination of the two components, as long as the total number of units ordered is within its capacity. Supplier capacities are as follows:

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Supplier	**Capacity**
1	600
2	1000
3	800

If the Edwards production plan for the next production period includes 1000 units of component 1 and 800 units of component 2, what purchases do you recommend? That is, how many units of each component should be ordered from each supplier? What is the total purchase cost for the components?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Let x_{ij} = units of component i purchased from supplier j

$$\text{Min} \quad 12x_{11} + 13x_{12} + 14x_{13} + 10x_{21} + 11x_{22} + 10x_{23}$$

s.t.

$$\begin{aligned}
x_{11} + x_{12} + x_{13} &= 1000 \\
x_{21} + x_{22} + x_{23} &= 800 \\
x_{11} + x_{21} &\le 600 \\
x_{12} + x_{22} &\le 1000 \\
x_{13} + x_{23} &\le 800 \\
x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} &\ge 0
\end{aligned}$$

Solution:

	Supplier 1	Supplier 2	Supplier 3
Component 1	600	400	0
Component 2	0	0	800

Purchase Cost: $20,400

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Production scheduling. The production manager for the Classic Boat Corporation must determine how many units of the Classic 21 model should be produced over the next four quarters. The company has a beginning inventory of 100 Classic 21 boats, and demand for the four quarters is 2000 units is quarter 1, 4000 units in quarter 2, 3000 units in quarter 3, and 1500 units in quarter 4. The firm has limited production capacity in each quarter. That is up to 4000 units can be produced in quarter 1, 3000 units in quarter 2, 2000 units in quarter 3, and 4000 units in quarter 4. Each boat that is held in inventory in quarters 1 and 2 incurs an inventory holding cost of $250 per unit; the holding cost for quarter 3 and 4 is $300 per unit. The production costs for the first quarter are $10,000 per unit; these costs are estimated to increase by 10 % each quarter because of increases in labor and material costs. Management has specified that the ending inventory for quarter 4 must be at least 500 boats.

a. Formulate a linear programming model that can be used to determine the production schedule that will minimize the total cost of meeting demand in each quarter subject to the production capacities in each quarter and also to the required ending inventory in quarter 4.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

b. Solve the linear program formulated in part (a); then develop a table that will show for each quarter the number of units to manufacture, the ending inventory, and the costs incurred.

c. Interpret each of the dual prices corresponding to the constraints developed to meet demand in each quarter. What advice would you give the production manager given these dual prices?

d. Interpret each of the dual prices corresponding to the production capacity in each quarter. What advice would you give the production manager given each of these dual prices?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

a. Let x_i = number of Classic 21 boats produced in Quarter i; $i = 1,2,3,4$

s_i = ending inventory of Classic 21 boats in Quarter i; $i = 1,2,3,4$

Min $10{,}000x_1 + 11{,}000x_2 + 12{,}100x_3 + 13{,}310x_4 + 250s_1 + 250s_2 + 300s_3 + 300s_4$

s.t.

$x_1 - s_1 = 1900$	Quarter 1 demand
$s_1 + x_2 - s_2 = 4000$	Quarter 2 demand
$s_2 + x_3 - s_3 = 3000$	Quarter 3 demand
$s_3 + x_4 - s_4 = 1500$	Quarter 4 demand
$s_4 \geq 500$	Ending inventory
$x_1 \leq 4000$	Quarter 1 capacity
$x_2 \leq 3000$	Quarter 2 capacity
$x_3 \leq 2000$	Quarter 3 capacity
$x_4 \leq 4000$	Quarter 4 capacity

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

b.

Quarter	Production	Ending Inventory	Cost
1	4000	2100	40,525,000
2	3000	1100	33,275,000
3	2000	100	24,230,000
4	1900	500	25,439,000
			$123,469,000

c. The dual prices tell us how much it would cost if demand were to increase by one additional unit. For example, in Quarter 2 the dual price is -12,760; thus, demand for one more boat in Quarter 2 will increase costs by $12,760.

d. The dual price of 0 for Quarter 4 tell us we have excess capacity in Quarter 4. The positive dual prices in Quarters 1-3 tell us how much increasing the production capacity will improve the objective function. For example, the dual price of $2510 for Quarter 1 tell us that if capacity is increased by 1 unit for this quarter, costs will go down $2510.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Staff scheduling. The Clark County Sheriff's Department schedules police officers for 8-hour shifts. The beginning times for the shifts are 8:00 A.M., noon, 4:00 P.M., 8:00 P.M., midnight, and 4:00 A.M. An officer beginning a shift at one of the above times works for the next 8 hours. During normal weekday operations, the number of officers needed varies depending on the time of day. The department staffing guidelines require the following minimum number of officers on duty:

Time of Day	Minimum Officers on Duty
8:00 A.M. – noon	5
Noon – 4:00 P.M.	6
4:00 A.M. – 8:00 P.M.	10
8:00 P.M. – Midnight	7
Midnight – 4:00 A.M.	4
4:00 A.M. – 8:00 A.M.	6

Determine the number of police officers that should be scheduled to begin the 8-hour shifts at each of the six times (8:00 A.M., noon, 4:00 P.M., 8:00 P.M., midnight, and 4:00 P.M.) such that the total number of officers required is minimized. (*Hint:* Let x_1 = the number of officers beginning work at 8:00 A.M. x_2 = the number of officers beginning work at noon, and so on.)

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Let x_1 = the number of officers shceduled to begin at 8:00 a.m.
x_2 = the number of officers shceduled to begin at noon
x_3 = the number of officers shceduled to begin at 4:00 p.m.
x_4 = the number of officers shceduled to begin at 8:00 p.m.
x_5 = the number of officers shceduled to begin at midnight
x_6 = the number of officers shceduled to begin at 4:00 a.m.

The objective function to minimize the number of officers required is as follows:

$$\text{Min} \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

The constraints require the total number of officers of duty each of the six four-hour periods to be at least equal to the minimum officer requirements. The constraints for the six four-hour periods are as follows:

Time of Day								
8:00 a.m. - noon	x_1					$+x_6$	$\geq$	5
noon to 4:00 p.m.	x_1	$+x_2$					$\geq$	6
4:00 p.m. - 8:00 p.m.		x_2	$+x_3$				$\geq$	10
8:00 p.m. - midnight			x_3	$+x_4$			$\geq$	7
midnight - 4:00 a.m.				x_4	$+x_5$		$\geq$	4
4:00 a.m. - 8:00 a.m.					x_5	$+x_6$	$\geq$	6

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Schedule 19 officers as follows:

$x_1 = 3$ begin at 8:00 a.m.
$x_2 = 3$ begin at noon
$x_3 = 7$ begin at 4:00 p.m.
$x_4 = 0$ begin at 8:00 p.m.
$x_5 = 4$ begin at midnight
$x_6 = 2$ begin at 4:00 a.m.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

The Grand Stand Oil Company produces regular-grade and premium-grade gasoline products for independent service stations in the southern United States. The Grand Stand refinery manufactures the gasoline products by blending three petroleum components. The gasolines are sold at different prices, and the petroleum components have different costs. The firm would like to determine how to mix or blend the three components into the two gasoline products in such a way as to maximize profits.

Data available show that the regular-grade gasoline can be sold for $0.50 per gallon and the premium-grade gasoline for $0.54 per gallon. For the current production planning period, Grand Stand can obtain the three petroleum components at the cost per gallon and in the quantities shown in Table 1.

The product specifications for the regular and premium gasolines restrict the amounts of each component that can be used in each gasoline product. The product specifications are shown in Table 2. Current commitments to distributors require Grand Stand to produce at least 10,000 gallons of regular-grade gasoline.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Table 1

Petroleum Component	Cost/Gallon	Maximum Available
1	$0.25	5,000 gallons
2	$0.30	10,000 gallons
3	$0.42	10,000 gallons

Table 2

Product	Specifications
Regular gasoline	At most 30% component 1
	At least 40% component 2
	At most 20% component 3
Premium gasoline	At least 25% component 1
	At most 40% component 2
	At least 30% component 3

The Grand Stand blending problem is to determine how many gallons of each component should be used in the regular-grade gasoline blend and how many should be used in the premium-grade gasoline blend. The optimal blending solution should maximize the firm's profit, subject to the constraints on the available petroleum supplies shown in Table 1, the product specifications in Table 2, and the required 10,000 gallons of regular-grade gasoline.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

We can use the following notation to define the decision variables:

$$x_{ij} = \text{gallons of component } i \text{ used in gasoline } j \text{ where } i = 1,2, \text{ or } 3$$

$$\text{for components } 1,2, \text{ or } 3, \text{ and } j = r \text{ if regular or } j = p \text{ if premium}$$

The six decision variables become:

$$x_{1r} = \text{gallons of component 1 in regular gasoline}$$

$$x_{2r} = \text{gallons of component 2 in regular gasoline}$$

$$x_{3r} = \text{gallons of component 3 in regular gasoline}$$

$$x_{1p} = \text{gallons of component 1 in premium gasoline}$$

$$x_{2p} = \text{gallons of component 2 in premium gasoline}$$

$$x_{3p} = \text{gallons of component 3 in premium gasoline}$$

Note that previously we have always used numbers as the subscripts for decision variables. Continuing this use, we could have let *j*=1 for regular gasoline and *j*=2 for premium gasoline. However, the use of the *r* and *p* subscripts is descriptive and will enable us to easily identify the gasoline product being referred to by the decision variable.

The total number of gallons of each type of gasoline produced is the sum of the number of gallons produced using each of the three petroleum components.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

That is:

Total Gallons produced

$$\text{Regular gasoline} = x_{1r} + x_{2r} + {}_{3r}$$

$$\text{Premium gasoline} = x_{1p} + x_{2p} + x_{3p}$$

Total Petroleum Component Usage

$$\text{Component 1} = x_{1r} + x_{1p}$$

$$\text{Component 2} = x_{2r} + x_{2p}$$

$$\text{Component 3} = x_{3r} + x_{3p}$$

The objective function of maximizing the profit contribution can be developed by identifying the difference between the total revenue from both gasolines and the total cost of the three petroleum components. By multiplying the \$0.50 per-gallon price by the total gallons of regular gasoline, the \$0.54 per-gallon price by the total gallons of premium gasoline, and the component cost per-gallon figures in Table 1 by the total gallons of each component used, the objective function can be written as follows:

$$\begin{aligned} \text{Max}\ & 0.50(x_{1r} + x_{2r} + x_{3r}) + 0.54(x_{1p} + x_{2p} + x_{3p}) \\ & - 0.25(x_{1r} + x_{1p}) - 0.30(x_{2r} + x_{2p}) - 0.42(x_{3r} + x_{3p}) \end{aligned}$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

By combining the terms, the objective function can be written as

$$\text{Max } 0.25x_{1r} + 0.20x_{2r} + 0.08x_{3r} + 0.29x_{1p} + 0.24x_{2p} + 0.12x_{3p}$$

The limitations on the availability of the three petroleum components can be expressed by the following three constraints:

$$x_{1r} + x_{1p} \le 5{,}000 \text{ Component 1}$$
$$x_{2r} + x_{2p} \le 10{,}000 \text{ Component 2}$$
$$x_{3r} + x_{3p} \le 10{,}000 \text{ Component 3}$$

Six constraints are now required to meet the product specifications stated in Table 2. The first specifications states that component 1 can account for at most 30% of the total gallons of regular gasoline produced. That is:

$$\frac{x_{1r}}{x_{1r} + x_{2r} + x_{3r}} \le 0.30$$

or

$$x_{1r} \le 0.30(x_{1r} + x_{2r} + x_{3r})$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Rewriting this constraint with the variables on the left - hand side and a constant on right - hand side, the first product specification constraint becomes

$$0.70x_{1r} - 0.30x_{2r} - 0.30x_{3r} \leq 0$$

The second product specification listed in Table 2 can be written as

$$\frac{x_{2r}}{x_{1r} + x_{2r} + x_{3r}} \geq 0.40$$

or

$$x_{2r} \geq 0.40(x_{1r} + x_{2r} + x_{3r})$$

and thus

$$-0.40x_{1r} + 0.60x_{2r} - 0.40x_{3r} \geq 0$$

Similarly, the four additional blending specifications shown in Table 2 can be written as

$$-0.20x_{1r} - 0.20x_{2r} + 0.80x_{3r} \leq 0$$

$$-0.75x_{1p} - 0.25x_{2p} - 0.25x_{3p} \geq 0$$

$$-0.40x_{1p} + 0.60x_{2p} - 0.40x_{3p} \leq 0$$

$$-0.30x_{1p} - 0.30x_{2p} + 0.70x_{3p} \geq 0$$

The constraint for at least 10,000 gallons of the regular - grade gasoline is written

$$x_{1r} + x_{2r} + x_{3r} \geq 10{,}000$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

The complete linear programming model with 6 decision variables and 10 constraints can be written as follows:

$$\text{Max} \quad 0.25x_{1r} + 0.20x_{2r} + 0.08x_{3r} + 0.29x_{1p} + 0.24x_{2p} + 0.12x_{3p}$$

s.t.

$$\begin{aligned}
x_{1r} + x_{1p} &\leq 5{,}000 \\
x_{2r} + x_{2p} &\leq 10{,}000 \\
x_{3r} + x_{3p} &\leq 10{,}000 \\
0.70x_{1r} - 0.30x_{2r} - 0.30x_{3r} &\leq 0 \\
-0.40x_{1r} + 0.60x_{2r} - 0.40x_{3r} &\geq 0 \\
-0.20x_{1r} - 0.20x_{2r} + 0.80x_{3r} &\leq 0 \\
0.75x_{1p} - 0.25x_{2p} - 0.25x_{3p} &\geq 0 \\
-0.40x_{1p} + 0.60x_{2p} - 0.40x_{3p} &\leq 0 \\
-0.30x_{1p} - 0.30x_{2p} + 0.70x_{3p} &\geq 0 \\
x_{1r} + x_{2r} + x_{3r} &\geq 10{,}000 \\
x_{1r}, x_{2r}, x_{3r}, x_{1p}, x_{2p}, x_{3p} &\geq 0
\end{aligned}$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Objective Function Value = 4650.000

Variable	Value	Reduced Costs
X1R	1250.000	0.000
X2R	6750.000	0.000
X3R	2000.000	0.000
X1P	3750.000	0.000
X2P	3250.847	0.000
X3P	7999.999	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	0.290
2	0.000	0.240
3	0.000	0.120
4	1750.000	0.000
5	2750.000	0.000
6	0.000	0.000
7	0.000	0.000
8	2750.000	0.000
9	3499.999	0.000
10	0.000	-0.040

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Production routing. Lurix Electronics manufactures two products that can be produced on two different production lines. Both products have their lowest production costs when produced on the more modern of the two production lines. However, the modern production line does not have the capacity to handle the total production. As a result, some production will have to be routed to the other production line. Shown below are the data for total production requirements, production line capacities, and production costs.

	Production Cost/Unit		
Product	**Modern Line**	**Old Line**	**Minimum Production Requirements**
1	\$3.00	\$5.00	500 units
2	\$2.50	\$4.00	700 units
Production line capacities	800	600	

Formulate a linear programming model that can be used to make the production routing decision. What is the recommended decision and the total cost?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Let x_{11} = Units of product 1 produced on Line 1

x_{12} = Units of product 1 produced on Line 2

x_{21} = Units of product 2 produced on Line 1

x_{22} = Units of product 2 produced on Line 2

$$\text{Min} \quad 3.00x_{11} + 5.00x_{12} + 2.50x_{21} + 4.00x_{22}$$

s.t.

$$\begin{aligned} x_{11} + x_{12} &\geq 500 \\ x_{21} + x_{22} &\geq 700 \\ x_{11} + x_{21} &\leq 800 \\ x_{12} + x_{22} &\leq 600 \\ x_{11}, x_{12}, x_{21}, x_{22} &\geq 0 \end{aligned}$$

Solution :

	Modern Line	Old Line
Product 1	500	0
Product 2	300	400

Total Cost : $3,850

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Overtime planning. The Hartman Company is trying to determine how much of each of two products should be produced over the coming planning period. Shown below is information concerning labor availability, labor utilization, and product profitability.

	Product (hours/unit)		
Department	**1**	**2**	**Labor-Hours Available**
A	1.00	0.35	100
B	0.30	0.20	36
C	0.20	0.50	50
Profit contribution/unit	$30.00	$15.00	-

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

a. Develop a linear programming model of the Hartman Company problem. Solve the model to determine the optimal production quantities of products 1 and 2.

b. In computing the per-unit profit contribution, Hartman does not deduct labor costs because they are considered fixed for the upcoming planning period. However, suppose overtime can be scheduled in some of the departments. Which departments would you recommend scheduling for overtime? How much would you be willing to pay per hour of overtime in each department?

c. Suppose that 10, 6 and 8 hours of overtimes may be scheduled in departments A, B and C, respectively. The cost per hour of overtime is $18 in department A, $22.50 in department B, and $12 in department C. Formulate a linear programming model that can be used to determine the optimal production quantities if overtime is made available. What are the optimal production quantities, and what is the revised total contribution profit? How much overtime do you recommend using in each department? What is the increase in the total contribution to profit if overtime is used?

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

a. Let $x_1 =$ units of product 1 produced
$x_2 =$ units of product 2 produced

$$\text{Max} \quad 30x_1 + 15x_2$$

s.t.

$$\begin{aligned} x_1 + 0.35x_2 &\le 100 \quad \text{Dept. A} \\ 0.30x_1 + 0.20x_2 &\le 36 \quad \text{Dept. B} \\ 0.20x_1 + 0.50x_2 &\le 50 \quad \text{Dept. C} \\ x_1, x_2 &\ge 0 \end{aligned}$$

Solution : $x_1 = 77.89, x_2 = 63.16$ Profit $= 3284.21$

b. The dual price for Dept. A is \$15.79, for Dept. B it is \$47.37, and for Dept. C it is \$0.00. Therefore, we would attempt to schedule overtime in Departments A and B. Assuming the current labor available is a sunk cost, we would be willing to pay up to \$15.79 per hour in Department A and up to \$47.37 in Department B.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

c. Let x_A = hours of overtime in Dept.A

x_B = hours of overtime in Dept.B

x_C = hours of overtime in Dept.C

$$\text{Max} \quad 30x_1 + 15x_2 - 18x_A - 22.5x_B - 12x_C$$

s.t.

$$\begin{aligned}
x_1 + 0.35x_2 - x_A &\le 100 \\
0.30x_1 + 0.20x_2 - x_B &\le 36 \\
0.20x_1 + 0.50x_2 - x_C &\le 50 \\
x_A &\le 10 \\
x_B &\le 6 \\
x_C &\le 8 \\
x_1, x_2, x_A, x_B, x_C &\ge 0
\end{aligned}$$

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

$$x_1 = 87.21$$
$$x_2 = 65.12$$
$$\text{Profit} = \$3341.34$$

Overtime	
Dept.A	10 hrs.
Dept.B	3.186 hrs.
Dept.C	0 hours

$$\text{Increase in Profit from overtime} = \$3341.34 - 3284.21 = \$57.13$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Quality assurance. Hilltop Coffee manufactures a coffee product by blending three types of coffee beans. The cost per pound and the available pounds of each bean are as follows:

Bean	Cost Per Pound	Available Pounds
1	$0.50	500
2	$0.70	600
3	$0.45	400

Consumer tests with coffee products were used to provide ratings on a 0-to-100 scale, with higher ratings indicating higher quality. Product quality standards for the blended coffee require a consumer rating for aroma to be at least 75 and a consumer rating for taste to be at least 80. The individual ratings of the aroma and taste for coffee made 100% of each bean are as follows:

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Bean	**Aroma Rating**	**Taste Rating**
1	75	86
2	85	88
3	60	75

It can be assumed that the aroma and taste attributes of the coffee blend will be a weighted average of the attributes of the beans used in the blend.

a. What is the minimum-cost blend that will meet the quality standards and provide 1000 pounds of the blended coffee product?
b. What is the cost per pound for the coffee blend?
c. Determine the aroma and taste ratings for the coffee blend.
d. If additional coffee were to be produced, what would be the expected cost per pound?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

a.

$x_1 = \text{pounds of bean 1}$

$x_2 = \text{pounds of bean 2}$

$x_3 = \text{pounds of bean 3}$

$$\text{Min} \quad 0.50x_1 + 0.70x_2 + 0.45x_3$$

s.t.

$$\frac{75x_1 + 85x_2 + 60x_3}{x_1 + x_2 + x_3} \geq 75$$

or $10x_2 - 15x_3 \geq 0$ Aroma

$$\frac{86x_1 + 88x_2 + 75x_3}{x_1 + x_2 + x_3} \geq 80$$

or $6x_1 + 8x_2 - 5x_3 \geq 0$ Taste

$x_1 \leq 500$ Bean 1

$x_2 \leq 600$ Bean 2

$x_3 \leq 400$ Bean 3

$x_1 + x_2 + x_3 = 1000$ 1000 Pounds

$x_1, x_2, x_3 \geq 0$

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

$$\text{Optimal Solution}: \; x_1 = 500, x_2 = 300, x_3 = 200 \quad \text{Cost}: \$550$$

b. $\text{Cost per pound} = \$550/1000 = \$0.55$

c. $\text{Surplus for aroma}: \; s_1 = 0; \text{ thus aroma rating} = 75$

$\text{Surplus for taste}: \quad s_2 = 4400; \text{ thus taste rating} = 80 + 4400/1000 \text{ lbs.} = 84.4$

d. $\text{Dual price} = -\$0.60.$ Extra coffee can be produced at a cost of \$0.60 per pound.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Blending problem. Ajax Fuels, Inc., is developing a new additive for airplane fuels. The additive is a mixture of three ingredients, A, B, and C. For proper performance, the total amount of additive (amount of A + amount of B + amount of C) must be at least 10 ounces per gallon of fuel. However, because of safety reasons, the amount of additive must not exceed 15 ounces per gallon of fuel. The mix or blend of the three ingredients is critical. At least 1 ounce of ingredient A must be used for every ounce of ingredient B. The amount of ingredient C must be greater than one-half the amount of ingredient A. If the cost per ounce for ingredients A, B, and C is $0.10, $0.03, and $0.09 respectively, find the minimum-cost mixture of A, B, and C for each gallon of airplane fuel.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Let $x_1 =$ amount of ingridient A

$x_2 =$ amount of ingredient B

$x_3 =$ amount of ingredient C

$$\text{Min} \quad 0.10x_1 + 0.03x_2 + 0.09x_3$$

s.t.

$$1x_1 + 1x_2 + 1x_3 \geq 10 \quad [1]$$

$$1x_1 + 1x_2 + 1x_3 \leq 15 \quad [2]$$

$$1x_1 \geq 1x_2$$

or

$$1x_1 - 1x_2 \geq 0 \quad [3]$$

$$1x_3 \geq 1/2x_1$$

or

$$-1/2x_1 + 1x_3 \geq 0 \quad [4]$$

$$x_1, x_2, x_3 \geq 0$$

Solution : $x_1 = 4, x_2 = 4, x_3 = 2$ Cost = \$0.70 per gallon.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Labor Planning. G. Kunz and Sons, Inc., manufactures two products used in the heavy equipment industry. Both products require manufacturing operations in two departments. Production time in hours and profit contribution figures for the two products are as follows:

		Labor-Hours	
Product	**Profit Per Unit**	**Dept. A**	**Dept. B**
1	$25	6	12
2	$20	8	10

For the coming production period, Kunz has available a total of 900 hours of labor that can be allocated to either of the two departments. Find the production plan and labor allocation (hours assigned in each department) that will maximize the total contribution to profit.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Let $x_1 = \text{units of product 1}$

$x_2 = \text{units of product 2}$

$b_1 = \text{labor - hours Dept.A}$

$b_2 = \text{labor - hours Dept.B}$

$$\text{Max} \quad 25x_1 + 20x_2 + 0b_1 + 0b_2$$

s.t.

$$\begin{aligned} 6x_1 + 8x_2 - 1b_1 \quad &= 0 \\ 12x_1 + 10x_2 - \quad 1b_2 &= 0 \\ 1b_1 + 1b_2 &\leq 900 \\ x_1, x_2, b_1, b_2 &\geq 0 \end{aligned}$$

Solution : $x_1 = 50, x_2 = 0, b_1 = 300, b_2 = 600$ Profit = \$1,250

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Make or buy. The Carson Stapler Manufacturing Company forecasts a 5000-unit demand for its Sure-Hold model during the next quarter. This stapler is assembled from three major components: base, staple cartridge, and handle. Until now Carson has manufactured all three components. However, the forecasts of 5000 units is a new high in sales volume, and it is doubtful that the firm will have sufficient production capacity to make all the components. The company is considering contracting a local firm to produce at least some of the components. The production time requirements per unit are as follows:

	Production Time(hours)			
Department	**Base**	**Cartridge**	**Handle**	**Total Available (hours)**
A	0.03	0.02	0.05	400
B	0.04	0.02	0.04	400
C	0.02	0.03	0.01	400

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

After considering the firm's overhead, material, and labor costs, the accounting department has determined the unit manufacturing cost for each component. These data, along with the purchase price quotations by the contracting firm, are as follows:

Component	**Manufacturing Cost**	**Purchase Cost**
Base	$0.75	$0.95
Cartridge	$0.40	$0.55
Handle	$1.10	$1.40

a. Determine the make-or-buy decision for Carson that will meet the 5000-unit demand at a minimum total cost. How many units of each component should be made, and how many purchased?

b. Which departments are limiting the manufacturing volume? If overtime could be considered at the additional cost of $3 per hour, which department(s) should be allocated the overtime? Explain.

c. Suppose that up to 80 hours of overtime could be scheduled in department A. What would you recommend?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Let $x_1 =$ number of units of the base manufactured
$x_2 =$ number of units of the cartridge manufactured
$x_3 =$ number of units of the handle manufactured
$x_4 =$ number of units of the base purchased
$x_5 =$ number of units of the cartridge purchased
$x_6 =$ number of units of the handle purchased

$$\begin{aligned}
\text{Min} \quad & 0.75x_1 + 0.40x_2 + 1.10x_3 + 0.95x_4 + 0.55x_5 + 1.40x_6 \\
\text{s.t.} \quad & \\
& 0.03x_1 + 0.02x_2 + 0.05x_3 \le 400 \quad \text{Dept.A} \\
& 0.04x_1 + 0.02x_2 + 0.04x_3 \le 400 \quad \text{Dept.B} \\
& 0.02x_1 + 0.03x_2 + 0.01x_3 \le 400 \quad \text{Dept.C} \\
& x_1 + x_4 = 5000 \\
& x_2 + x_5 = 5000 \\
& x_3 + x_6 = 5000 \\
& x_1, x_2, x_3, x_4, x_5, x_6 \ge 0
\end{aligned}$$

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

a. $x_1 = 3{,}750$
$x_2 = 5{,}000$
$x_3 = 3{,}750$
$x_4 = 1{,}250$ purchase
$x_5 = 0$
$x_6 = 1{,}250$ purchase Cost $= \$11{,}875$

b. Departments A and B are at capacity. The dual prices show additional hours in A are worth \$5 and additional hours in B are worth \$1.25. Based on the added \$3 per hour cost, we should only consider adding hours in Department A (Net Value = \$5 - \$3 = \$2 per hour).

c. All 80 hours cannot be used in Department A. In fact only 25 hours can be added before a change in solution occurs.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

New solution:

$x_1 = 2{,}500$

$x_2 = 5{,}000$

$x_3 = 5{,}000$ Total Cost $11,750

$x_4 = 1{,}250$ plus 25 hrs.@$3/hr. 75

$x_5 = 0$ $11,825

$x_6 = 0$

At this point, only Department B is at capacity. Overtime hours in this department now have a potential value of $5 per hour.

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Seastrand Oil Company produces two grades of gasoline; regular and high octane. Both types of gasolines are produced by blending two types of crude oil. Although both types of crude oil contain the two important ingredients required to produce both gasolines, the percentage of important ingredients in each type of crude oil differs, as well as the cost per gallon. The percentage of ingredients A and B in each type of crude oil, and the cost per gallon, are shown below:

Type of Crude Oil	**Cost**	**Ingredient A**	**Ingredient B**
1	$0.10	20%	60%
2	$0.15	50%	30%

Crude 1 is 60% ingredient B

Each gallon of regular must contain at least 40% of A, whereas each gallon of high octane can contain at most 50% of B. Daily demand for regular octane gasoline is 800,000 gallons, and daily demand for high octane is 500,000 gallons. How many gallons of each type of crude oil should be used in regular and high octane gasoline in order to satisfy daily demand at minimum cost?

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Defining the Decision Variables

		Product	
		Regular	High Octane
Raw Material	Crude 1	x_{11}	x_{12}
	Crude 2	x_{21}	x_{22}

x_{11} = number of gallons of crude 1 used to make regular
x_{12} = number of gallons of crude 1 used to make high octane
x_{21} = number of gallons of crude 2 used to make regular
x_{22} = number of gallons of crude 2 used to make high octane

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Each Gallon of Regular to have at least 40% of Ingredient A

Note that the amount of regular produced is

$$x_{11} + x_{21}$$

The minimum amount of A required for regular is

$$.4(x_{11} + x_{21})$$

The total amount of A in $(x_{11} + x_{21})$ gallons of regular gasoline is

$$.2x_{11} + .5x_{21}$$

Thus the constraint that requires that each gallon of regular must contain at least 40% of A can be written

$$.2x_{11} + .5x_{21} \geq .4(x_{11} + x_{21})$$
$$-.2x_{11} + .1x_{21} \geq 0$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Each Gallon of High Octane to have at Most 50% of Ingredient B

Note that the amount of high octane produced is

$$x_{12} + x_{22}$$

The maximum amount of B in high octane is

$$.5(x_{12} + x_{22})$$

The total amount of B in $(x_{12} + x_{22})$ gallons of high octane gasoline is

$$.6x_{12} + .3x_{22}$$

Thus the constraint that requires that each gallon of high octane must contain at most 40% of ingredient B is

$$.6x_{12} + .3x_{22} \leq .5(x_{12} + x_{22})$$

$$.1x_{12} - .2x_{22} \leq 0$$

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Demand for Regular

$$x_{11} + x_{21} \geq 800{,}000$$

Demand for High Octane

$$x_{12} + x_{22} \geq 500{,}000$$

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

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LINEAR PROGRAMMING PROBLEM

min .1X11 +.1X12 +.15x21 +.15x22

     S.T.

        1)   -.2X11+.1X21 > 0
        2)   .1X12-.2X22 < 0
        3)   X11+X21 > 800000
        4)   X12+X22 > 500000
```

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ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

Objective Function Value = 165000.016

Variable	Value	Reduced Costs
X11	266666.656	0.000
X12	333333.344	0.000
X21	533333.375	0.000
x22	166666.672	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	-0.167
2	0.000	0.167
3	0.000	-0.133
4	0.000	-0.117

ΕΦΑΡΜΟΓΕΣ ΓΠ ΣΤΗΝ ΠΑΡΑΓΩΓΗ
LP PRODUCTION APPLICATIONS

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X11	-0.300	0.100	0.150
X12	-0.075	0.100	0.150
X21	0.100	0.150	No Upper Limit
X22	0.100	0.150	No Upper Limit

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	-160000.016	0.000	80000.000
2	-100000.008	0.000	50000.004
3	0.000	800000.000	No Upper Limit
4	0.000	500000.000	No Upper Limit