ΠΡΟΒΛΗΜΑΤΑ ΡΟΗΣ ΔΙΚΤΥΩΝ
NETWORK FLOW PROBLEMS

- **Πρόβλημα Μεταφοράς**
    - » **Transportation Problem**
- **Πρόβλημα Εκχώρησης**
    - » **Assignment Problem**
- **Πρόβλημα Μεταφόρτωσης**
    - » **Transshipment Problem**

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Let us illustrate by considering the transportation problem faced by Foster Generators. This problem involves the transportation of a product from three plants to four distribution centers. Foster Generators has production operations in Cleveland, Ohio; Bedford, Indiana; and York, Pennsylvania. Production capacities for these plants over the next 3-month planning period for one type of generator are as follows:

Origin	**Plant**	**3-Month Production Capacity (units)**
1	Cleveland	5,000
2	Bedford	6,000
3	York	2,500
	Total	13,500

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

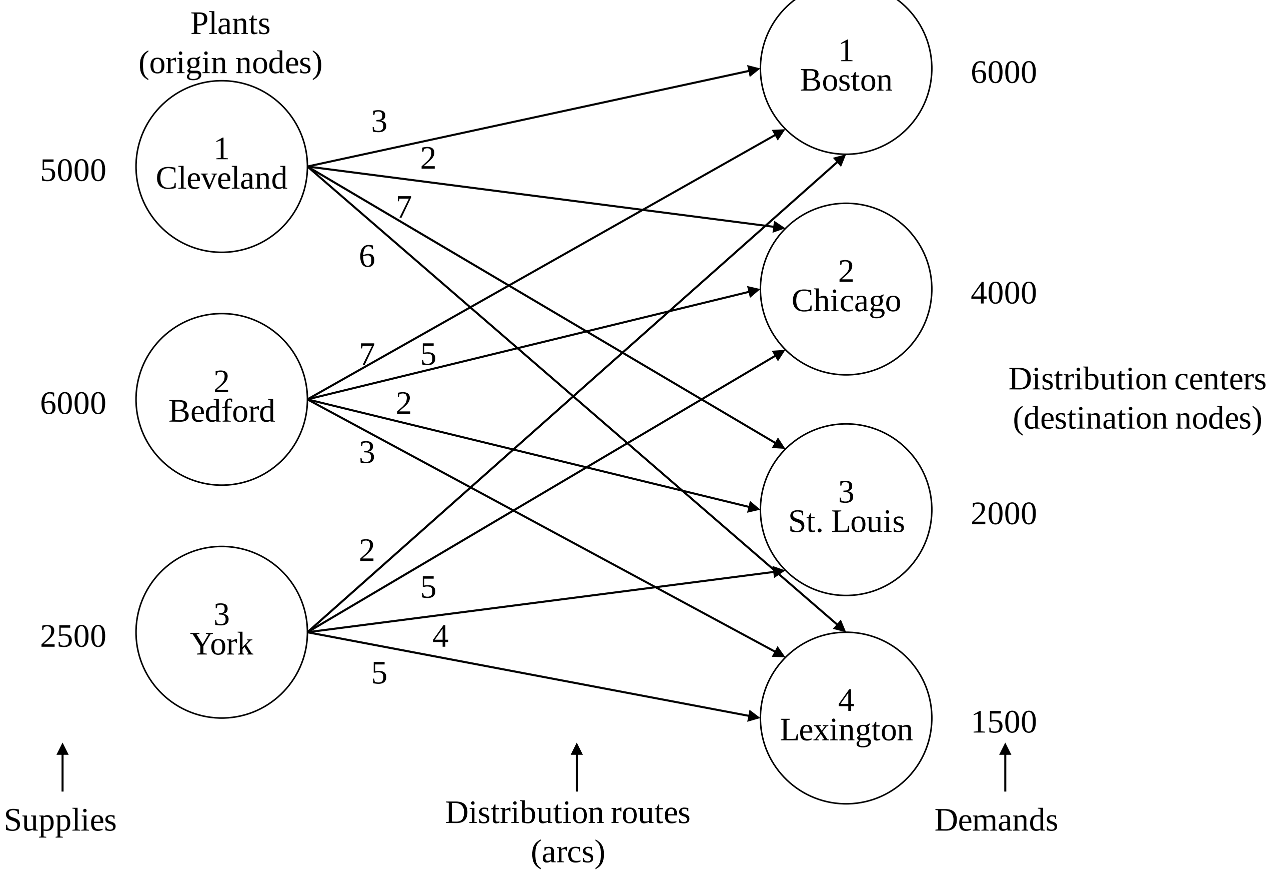
The firm distributes its generators through four regional distribution centers located in Boston, Chicago, St. Louis, and Lexington; the 3-month forecast of demand for the distribution centers is as follows:

Destination	**Distribution Center**	**3-Month Demand Forecast (units)**
1	Boston	6,000
2	Chicago	4,000
3	St. Louis	2,000
4	Lexington	1,500
	Total	13,500

	Destination			
Origin	Boston	Chicago	St. Louis	Lexington
Cleveland	3	2	7	6
Bedford	7	5	2	3
York	2	5	4	5

Transportation Cost Table

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM



ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

$$x_{ij} = \text{number of units shipped from origin } i \text{ to destination } j$$
$$\text{where } i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n$$

Note that there is one decision variable for each arc in the Figure. Using this notation, $x_{24} = 500$ would correspond to shipping 500 units from Bedford (origin 2) to Lexington (destination 4).

Since the objective of the transportation problem is to minimize the total transportation cost, we can use the cost data in the Table or on the arcs in the Figure to develop the following cost expressions:

$$\begin{aligned}
&\text{Transportation costs for} \\
&\text{units shipped from Cleveland} = 3x_{11} + 2x_{12} + 7x_{13} + 6x_{14} \\
&\text{Transportation costs for} \\
&\text{units shipped from Bedford} \quad = 7x_{21} + 5x_{22} + 2x_{23} + 3x_{24} \\
&\text{Transportation costs for} \\
&\text{units shipped from York} \qquad = 2x_{31} + 5x_{32} + 4x_{33} + 5x_{34}
\end{aligned}$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

The sum of the above expressions provides the objective function showing the total transportation cost for Foster Generators.

Transportation problems need constraints because each origin has a limited supply and each destination has a specific demand. We will consider the supply constraints first. The capacity at the Cleveland plant is 5000 units. With the total number of units shipped from the Cleveland plant expressed as $x_{11} + x_{12} + x_{13} + x_{14}$ the supply constraint for the Cleveland plant can be written as

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 5000 \text{ Cleveland supply}$$

With three origins (plants), the Foster transportation problem has three supply constraints. Given the capacity of 6000 units at the Bedford plant and 2500 units at the York plant, the two additional supply constraints are as follows:

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 6000 \text{ Bedford supply}$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 2500 \text{ York supply}$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ

TRANSPORTATION PROBLEM

With the four distribution centers as the destinations, the following four demand constraints are needed to ensure that destination demands will be satisfied:

$$
\begin{aligned}
x_{11} + x_{21} + x_{31} &= 6000 \text{ Boston demand} \\
x_{12} + x_{22} + x_{32} &= 4000 \text{ Chicago demand} \\
x_{13} + x_{23} + x_{33} &= 2000 \text{ St. Louis demand} \\
x_{14} + x_{24} + x_{34} &= 1500 \text{ Lexington demand}
\end{aligned}
$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

$$\text{Min}\quad 3x_{11}+2x_{12}+7x_{13}+6x_{14}+7x_{21}+5x_{22}+2x_{23}+3x_{24}+2x_{31}+5x_{32}+4x_{33}+5x_{34}$$

s.t.

$$\begin{array}{llllllllllllll}
x_{11}+ & x_{12}+ & x_{13}+ & x_{14} & & & & & & & & & \le 5000 \\
 & & & & x_{21}+ & x_{22}+ & x_{23}+ & x_{24} & & & & & \le 6000 \\
 & & & & & & & & x_{31}+ & x_{32}+ & x_{33}+ & x_{34} & \le 2500 \\
x_{11}+ & & & & x_{21}+ & & & & x_{31} & & & & = 6000 \\
 & x_{12}+ & & & & x_{22}+ & & & & x_{32} & & & = 4000 \\
 & & x_{13}+ & & & & x_{23}+ & & & & x_{33} & & = 2000 \\
 & & & x_{14}+ & & & & x_{24}+ & & & & x_{34} & = 1500
\end{array}$$

$$x_{ij} \ge 0 \qquad \text{for } i=1,2,3 \text{ and } j=1,2,3,4$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

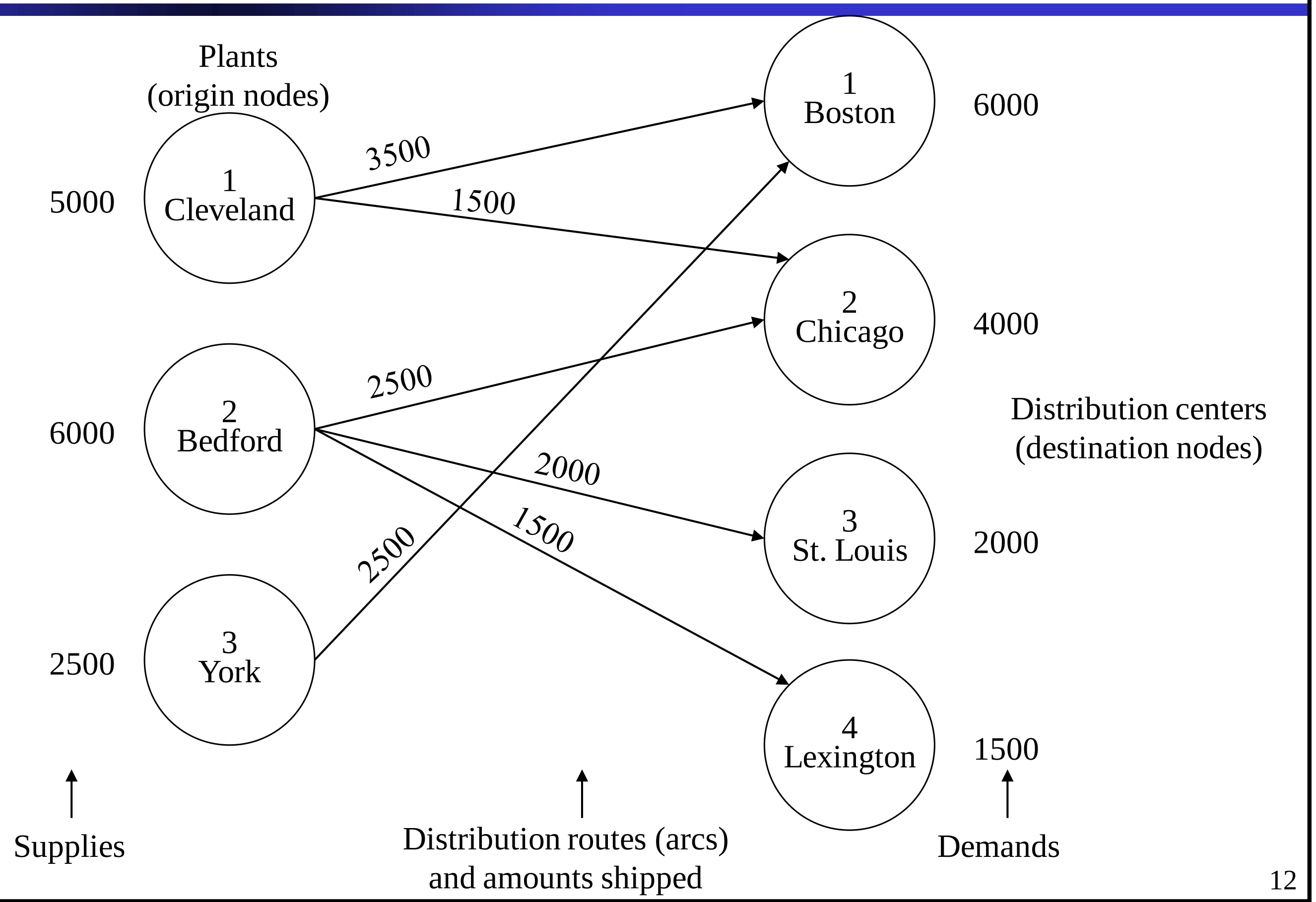
```
Objective Function Value =        39500.000

      Variable             Value             Reduced Costs
   --------------     ---------------     ------------------
         X11                3500.000                  0.000
         X12                1500.000                  0.000
         X13                   0.000                  8.000
         X14                   0.000                  6.000
         X21                   0.000                  1.000
         X22                2500.000                  0.000
         X23                2000.000                  0.000
         X24                1500.000                  0.000
         X31                2500.000                  0.000
         X32                   0.000                  4.000
         X33                   0.000                  6.000
         X34                   0.000                  6.000
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Route				
From	**To**	**Units Shipped**	**Per-Unit Cost**	**Total Cost**
Cleveland	Boston	3500	$3	$10,500
Cleveland	Chicago	1500	$2	3,000
Bedford	Chicago	2500	$5	12,500
Bedford	St. Louis	2000	$2	4,000
Bedford	Lexington	1500	$3	4,500
York	Boston	2500	$2	5,000
				$39,500

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM



ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

To show the general linear programming model of the transportation problem, we use the following notation:

$i =$ index for origins, $i = 1,2,\ldots,m$

$j =$ index for destinations, $j = 1,2,\ldots,n$

$x_{ij} =$ number of units shipped from origin i to destination j

$c_{ij} =$ cost per unit of shipping from origin i to destination j

$s_i =$ supply or capacity in units at origin i

$d_j =$ demand in units at destination j

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

The general linear programming model of the *m*-origin, *n*-destination transportation problem is

$$\text{Min} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

s.t.

$$\sum_{j=1}^{n} x_{ij} \le s_i \qquad i = 1,2,\ldots,m \quad \text{Supply}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1,2,\ldots,n \quad \text{Demand}$$

$$x_{ij} \ge 0 \qquad \text{for all } i \text{ and } j$$

As mentioned above, additional constraints of the form $x_{ij} \le L_{ij}$ can be added if the route from origin *i* to destination *j* has capacity L_{ij}. A transportation problem that includes constraints of this type is called a *capacitated transportation problem*.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

- **ΠΑΡΑΛΛΑΓΕΣ ΠΡΟΒΛΗΜΑΤΟΣ** (PROBLEM VARIATIONS)
  - **Ανιση συνολική ζήτηση και προσφορά/προμήθεια** (Total supply not equal to total demand).
  - **Μεγιστοποίηση αντί ελαχιστοποίηση αντικειμενικής συνάρτησης** (Maximization objective function rather than minimization).
  - **Μέγιστη και ελάχιστη δυναμικότητα στις διαδρομές (**Route capacities or route minimums).

    e.g. $400 \leq x_{31} \leq 1000$
  - **Μη αποδεκτές διαδρομές** (Unacceptable routes).

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

- **Ανιση συνολική ζήτηση και προσφορά/προμήθεια** (Total supply not equal to total demand).
  - Εαν η συνολική παραγωγή/προσφορά/προμήθεια υπερβαίνει της συνολικής ζήτησης, καμιά αλλαγή δεν είναι απαραίτητο να γίνει στη διατύπωση του προβλήματος σαν γραμμικό πρόγραμμα.
    - If total supply exceeds total demand, no modifications on the linear programming formulation are necessary.
  - Η επιπλέον παραγωγή θα εμφανιστεί σαν χαλαρότητα στη λύση.
    - Excess supply will appear as slack in the solution.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

- **Ανιση συνολική ζήτηση και προσφορά/προμήθεια** (Total supply not equal to total demand).
  - Εαν η συνολική παραγωγή είναι μικρότερη της συνολικής ζήτησης, τότε διαφοροποιούμε το δίκτυο με ένα νέο κόμβο (dummy node) του οποίου η παραγωγή είναι ίση με τη διαφορά της συνολικής ζήτησης και συνολικής παραγωγής.
    - If total supply is less than total demand we modify the network by introducing a dummy origin with supply equal to the difference between total demand and total supply.
  - Αναθέτουμε μηδενικό κόστος μεταφοράς στα τόξα που αρχίζουν απο τον κόμβο dummy.
    - A zero per-unit cost is assigned to the arcs leaving the dummy node.
  - Στη βέλτιστη λύση, προορισμοί που εφοδιάζονται από τον κόμβο dummy είναι προορισμοί των οποίων η ζήτηση δεν θα ικανοποιηθεί.
    - At the optimal solution, destinations receiving shipments from the dummy node will be destinations experiencing a shortfall or unsatisfied demand.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Specialized module for solving the transportation problem in the software Management Scientist.

```
TRANSPORTATION PROBLEM
**********************

OBJECTIVE: MINIMIZATION

SUMMARY OF ORIGIN SUPPLIES
**************************

     ORIGIN       SUPPLY
     ------       ------
        1          5000
        2          6000
        3          2500
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

```
SUMMARY OF DESTINATION DEMANDS
******************************

  DESTINATION    DEMAND
  -----------    ------
        1         6000
        2         4000
        3         2000
        4         1500

SUMMARY OF UNIT COST OR REVENUE DATA
************************************

  FROM          TO DESTINATION
 ORIGIN    1      2      3      4
 ------  -----  -----  -----  -----
    1        3      2      7      6
    2        7      5      2      3
    3        2      5      4      5
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

```
     OPTIMAL TRANSPORTATION SCHEDULE
     *******************************

   SHIP
   FROM            TO DESTINATION
  ORIGIN     1       2       3       4
  ------   -----   -----   -----   -----
     1      3500    1500       0       0
     2         0    2500    2000    1500
     3      2500       0       0       0

      TOTAL TRASNPORTATION COST OR REVENUE IS 39500
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

```
TRANSPORTATION PROBLEM
**********************

OBJECTIVE: MINIMIZATION

SUMMARY OF ORIGIN SUPPLIES
**************************

     ORIGIN      SUPPLY
     ------      ------
        1         6000
        2         6000
        3         2500
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

```
SUMMARY OF DESTINATION DEMANDS
******************************

  DESTINATION    DEMAND
  -----------    ------
        1         6000
        2         4000
        3         2000
        4         1500

SUMMARY OF UNIT COST OR REVENUE DATA
************************************

  FROM          TO DESTINATION
 ORIGIN    1      2      3      4
 ------  -----  -----  -----  -----
    1        3      2      7      6
    2        7      5      2      3
    3        2      5      4      5
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

```
      OPTIMAL TRANSPORTATION SCHEDULE
      *******************************

   SHIP
   FROM           TO DESTINATION
  ORIGIN    1      2      3      4
  ------  -----  -----  -----  -----
     1     3500   2500      0      0
     2        0   1500   2000   1500
     3     2500      0      0      0

       TOTAL TRANSPORTATION COST OR REVENUE IS 36500
       NOTE: THE TOTAL SUPPLY EXCEEDS THE TOTAL DEMAND BY
       1000

          ORIGIN      EXCESS SUPPLY
          ------      -------------
              2               1000
```

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ

ASSIGNMENT PROBLEM

As an illustration of the assignment problem, let us consider the case of Fowle Marketing Research which has just received requests for market research studies from three new clients. The company is faced with the task of assigning project leaders (agents) to each of these three new research studies (tasks). Currently, three individuals are free from other commitments and available for the project leader assignments. Fowle's management realizes, however, that the time required to complete each study will depend on the experience and ability of the project leader assigned to the study. Since the three projects have been judged to have approximately the same priority, the company would like to assign project leaders such that the total number of days required to complete all three projects is minimized. If a project leader is to be assigned to one and only one client, what assignments should be made?

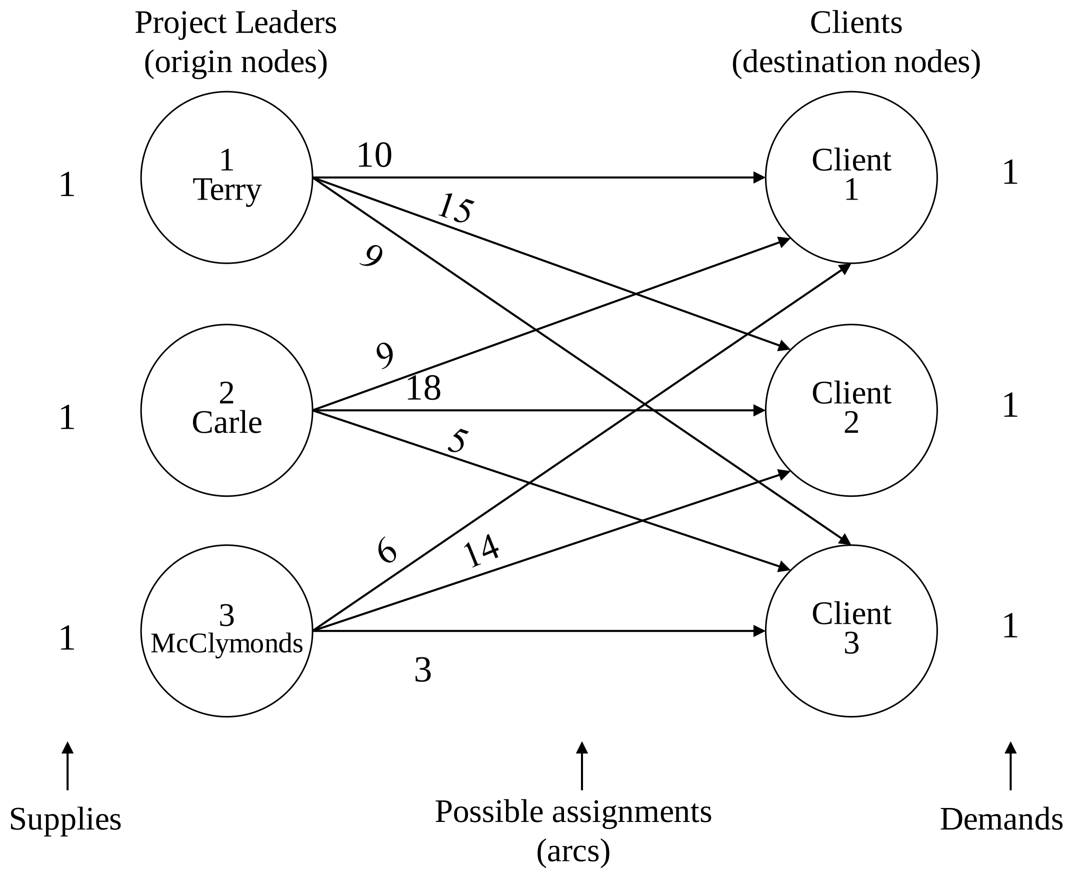
To answer the assignment question, Fowle's management must first consider all possible project leader-client assignments and then estimate the corresponding project completion times. With three project leaders and three clients, there are nine possible assignment alternatives.

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

The alternatives and the estimated project completion times in days are summarized in the following Table. Using these data, we see that Terry would require 10 days to complete client’s 1 project, while Carle would require 9 days for the same project. Similar completion time statements can be made about the other possible assignments.

	Client		
Project Leader	**1**	**2**	**3**
1. Terry	10	15	9
2. Carle	9	18	5
3. McClymonds	6	14	3

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM



ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

$$x_{ij} = \begin{Bmatrix} 1 \text{ if project leader } i \text{ is assigned to client } j \\ 0 \text{ otherwise} \end{Bmatrix}$$

$$\text{where } i = 1,2,3, \text{ and } j = 1,2,3$$

Using this notation, $x_{21} = 1$ and $x_{31} = 0$ would tell us that project leader 2 (Carle) is assigned to client 1, and project leader 3 (McClymonds) is not assigned to client 1.

Following this notation and using the completion time data in the Table, we can develop the following completion time expressions:

$$\text{Days required for Terry's assignment} = 10x_{11} + 15x_{12} + 9x_{13}$$

$$\text{Days required for Carle's assignment} = 9x_{21} + 18x_{22} + 5x_{23}$$

$$\text{Days required for McClymonds's assignment} = 6x_{31} + 14x_{32} + 3x_{33}$$

The sum of the completion times for the three project leaders will provide the total days required to complete the three assignments. Thus, the objective function is:

$$\text{Min } 10x_{11} + 15x_{12} + 9x_{13} + 9x_{21} + 18x_{22} + 5x_{23} + 6x_{31} + 14x_{32} + 3x_{33}$$

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

The constraints for the assignment problem reflect the conditions that each project leader can be assigned to at most one client and that each client must have one assigned project leader. These constraints are written as follows:

$$
\begin{aligned}
&x_{11}+x_{12}+x_{13}\leq 1 \quad \text{Terry's assignment}\\
&x_{21}+x_{22}+x_{23}\leq 1 \quad \text{Carle's assignment}\\
&x_{31}+x_{32}+x_{33}\leq 1 \quad \text{McClymonds's assignment}\\
&x_{11}+x_{21}+x_{31}=1 \quad \text{Client 1}\\
&x_{12}+x_{22}+x_{32}=1 \quad \text{Client 2}\\
&x_{13}+x_{23}+x_{33}=1 \quad \text{Client 3}
\end{aligned}
$$

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

$$\text{Min } \; 10x_{11}+15x_{12}+9x_{13}+9x_{21}+18x_{22}+5x_{23}+6x_{31}+14x_{32}+3x_{33}$$

s.t.

$$\begin{array}{lllllllllr}
x_{11}+ & x_{12}+ & x_{13} & & & & & & & \le 1 \\
 & & & x_{21}+ & x_{22}+ & x_{23} & & & & \le 1 \\
 & & & & & & x_{31}+ & x_{32}+ & x_{33} & \le 1 \\
x_{11}+ & & & x_{21}+ & & & x_{31} & & & =1 \\
 & x_{12}+ & & & x_{22}+ & & & x_{32} & & =1 \\
 & & x_{13}+ & & & x_{23}+ & & & x_{33} & =1
\end{array}$$

$x_{ij} \ge 0$ for $i=1,2,3,$ and $j=1,2,3$

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

```
Objective Function Value =       26.000

      Variable             Value              Reduced Costs
   --------------      ---------------      ------------------
         X11                0.000                 0.000
         X12                1.000                 0.000
         X13                0.000                 3.000
         X21                0.000                 0.000
         X22                0.000                 4.000
         X23                1.000                 0.000
         X31                1.000                 0.000
         X32                0.000                 3.000
         X33                0.000                 1.000
```

Project Leader	**Assigned Client**	**Days**
Terry	2	15
Carle	3	5
McClymonds	1	6
	Total	26

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

The general assignment problem involves *m* agents and *n* tasks. If we let $x_{ij} = 1 \text{ or } 0$ according to whether agent *i* is assigned to task *j* or not, and if c_{ij} denotes the cost of assigning agent *i* to task *j*, then we can write the general assignment model as follows:

$$\text{Min} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

s.t.

$$\sum_{j=1}^{n} x_{ij} \leq 1 \qquad i = 1,2,\ldots,m \quad \text{Agents}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1,2,\ldots,n \quad \text{Tasks}$$

$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j$$

If the number of tasks *n* exceeds the number of agents *m* (i.e. if demand exceeds supply), *n* - *m* dummy agents must be included for us to obtain a feasible solution.

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Multiple Assignments :

e.g. $x_{11} + x_{12} + x_{13} \leq 1 \Rightarrow x_{11} + x_{12} + x_{13} \leq 2$

In general, if a_i denotes the upper limit for the number of tasks to which agent i can be assigned, we write the agent constraints as :

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \ i = 1,2,\ldots,m$$

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Specialized module for solving the assignment problem in the software Management Scientist.

```
   ASSIGNMENT PROBLEM
   ******************

   OBJECTIVE: MINIMIZATION

   SUMMARY OF UNIT COST OR REVENUE DATA
   ************************************

                 TASK
AGENT       1      2      3
-----     -----  -----  -----
   1         10     15      9
   2          9     18      5
   3          6     14      3
```

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ

ASSIGNMENT PROBLEM

```
  OPTIMAL ASSIGNMENTS            COST/REVENUE
  *******************            ************

ASSIGN AGENT 3 TO TASK 1                  6
ASSIGN AGENT 1 TO TASK 2                 15
ASSIGN AGENT 2 TO TASK 3                  5
                                  ----------
  TOTAL COST/REVENUE                     26
```

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

```
   ASSIGNMENT PROBLEM
   ******************

   OBJECTIVE: MINIMIZATION

   SUMMARY OF UNIT COST OR REVENUE DATA
   ************************************

                 TASK
AGENT       1      2      3
-----     -----  -----  -----
   1         10     15      9
   2          9     18      5
```

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

```
  OPTIMAL ASSIGNMENTS          COST/REVENUE
  *******************          ************

ASSIGN AGENT 1 TO TASK 1              10
ASSIGN AGENT 2 TO TASK 3               5
                                 ----------
  TOTAL COST/REVENUE                  15

NOTE:   THE NUMBER OF TASKS EXCEEDS THE NUMBER OF
AGENTS
        1 TASK REMAINS UNASSIGNED.
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

Let us consider the transshipment problem faced by Ryan Electronics. Ryan is an electronics company with production facilities located in Denver and Atlanta. Components produced at either facility may be shipped to either of the firm's regional warehouses, which are located in Kansas City and Louisville. From the regional warehouses, the firm supplies retail outlets in Detroit, Miami, Dallas, and New Orleans.

The key features of the problem are shown in the network model depicted in the Figure that follows. Note that the supply at each plant and the demand at each retail outlet are shown in the left and right margins, respectively. Nodes 1 and 2 are the origin nodes; nodes 3 and 4 are the transshipment nodes; and nodes 5,6,7, and 8 are the destination nodes. The transportation cost per unit for each distribution route is shown in the following Table and on the arcs of the network model.

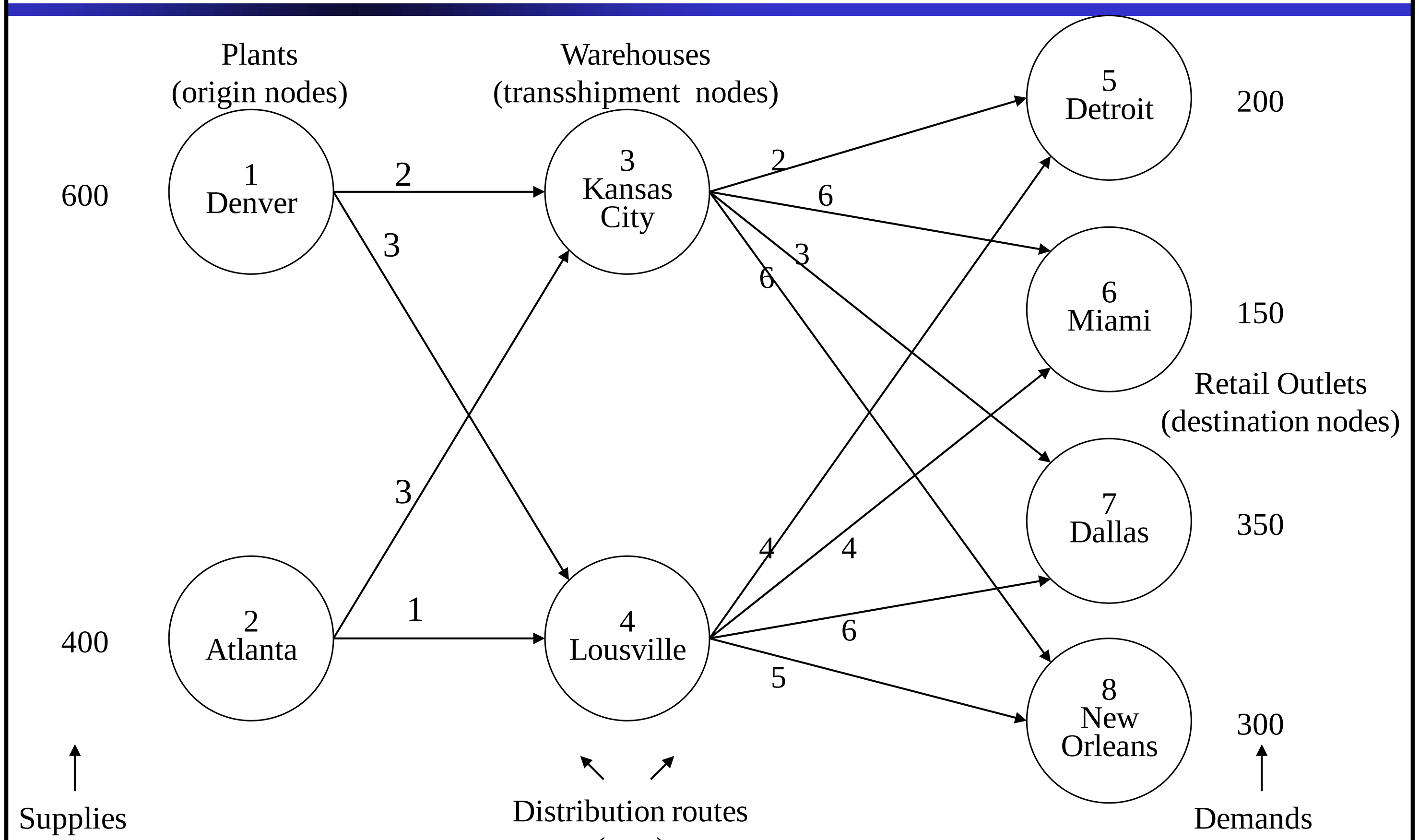
ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ

TRANSSHIPMENT PROBLEM

	Warehouse	
Plant	Kansas City	Louisville
Denver	2	3
Atlanta	3	1

	Retail Outlets			
Warehouse	Detroit	Miami	Dallas	New Orleans
Kansas City	2	6	3	6
Louisville	4	4	6	5

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM



ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

As with the transportation and assignment problems, it is easy to formulate a linear programming model of the transshipment problem given the network representation. Again, **we need a constraint for each node and a variable for each arc**. Let x_{ij} denote the number of units shipped from node *i* to node *j*. For example, x_{13} denotes the number of units shipped from the Denver plant to the Kansas City warehouse, x_{14} denotes the number of units shipped from the Denver plant to the Louisville warehouse, and so on. Since the supply at the Denver plant is 600 units, the amount shipped out of Denver plant must be less than or equal to 600. Mathematically, this supply constraint is written

$$x_{13} + x_{14} \le 600$$

Similarly, for the Atlanta plant we have

$$x_{23} + x_{24} \le 400$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

Let us now consider how we must write the constraints corresponding to the two transshipment nodes. For node 3 (the Kansas City warehouse), we must guarantee that the number of units shipped out must equal the number of units shipped into the warehouse. Since

$$\text{Number of units shipped out of node } 3 = x_{35} + x_{36} + x_{37} + x_{38}$$

and

$$\text{Number of units shipped into node } 3 = x_{13} + x_{23}$$

we obtain

$$x_{35} + x_{36} + x_{37} + x_{38} = x_{13} + x_{23}$$

Placing all the variables on the left - hand side of the expression enables us to write the constraint corresponding to node 3 as

$$-x_{13} - x_{23} + x_{35} + x_{36} + x_{37} + x_{38} = 0$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

In a similar manner, the constraint corresponding to node 4 is

$$-x_{14}-x_{24}+x_{45}+x_{46}+x_{47}+x_{48}=0$$

To develop the constraints associated with the destination nodes, we recognise that for each node the amount shipped to the destination must equal the demand. For example, to satisfy the demand for 200 units at node 5(the Detroit retail outlet) we can write

$$x_{35}+x_{45}=200$$

Similarly, for nodes 6, 7, and 8, we have the following constraints:

$$x_{36}+x_{46}=150$$

$$x_{37}+x_{47}=350$$

$$x_{38}+x_{48}=300$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

$$\text{Min}\ \ 2x_{13}+3x_{14}+3x_{23}+1x_{24}+2x_{35}+6x_{36}+3x_{37}+6x_{38}+4x_{45}+4x_{46}+6x_{47}+5x_{48}$$

s.t.

$$\left.\begin{array}{llllllllllllll}
x_{13}+ & x_{14} & & & & & & & & & & & & \le 600 \\
 & & x_{23}+ & x_{24} & & & & & & & & & & \le 400
\end{array}\right\}\ 1$$

$$\left.\begin{array}{llllllllllllll}
-x_{13}- & & x_{23}+ & & x_{35}+ & x_{36}+ & x_{37}+ & x_{38} & & & & & & = \ \ 0 \\
 & -x_{14}- & & x_{24}+ & & & & & x_{45}+ & x_{46}+ & x_{47}+ & x_{48} & & = \ \ 0
\end{array}\right\}\ 2$$

$$\left.\begin{array}{llllllllllllll}
x_{35}+ & & & & x_{45} & & & & = 200 \\
 & x_{36}+ & & & & x_{46} & & & = 150 \\
 & & x_{37}+ & & & & x_{47} & & = 350 \\
 & & & x_{38}+ & & & & x_{48} & = 300
\end{array}\right\}\ 3$$

$x_{ij} \ge 0$ for all i and j

1. Origin node constraints
2. Transshipment node constraints
3. Destination node constraints

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

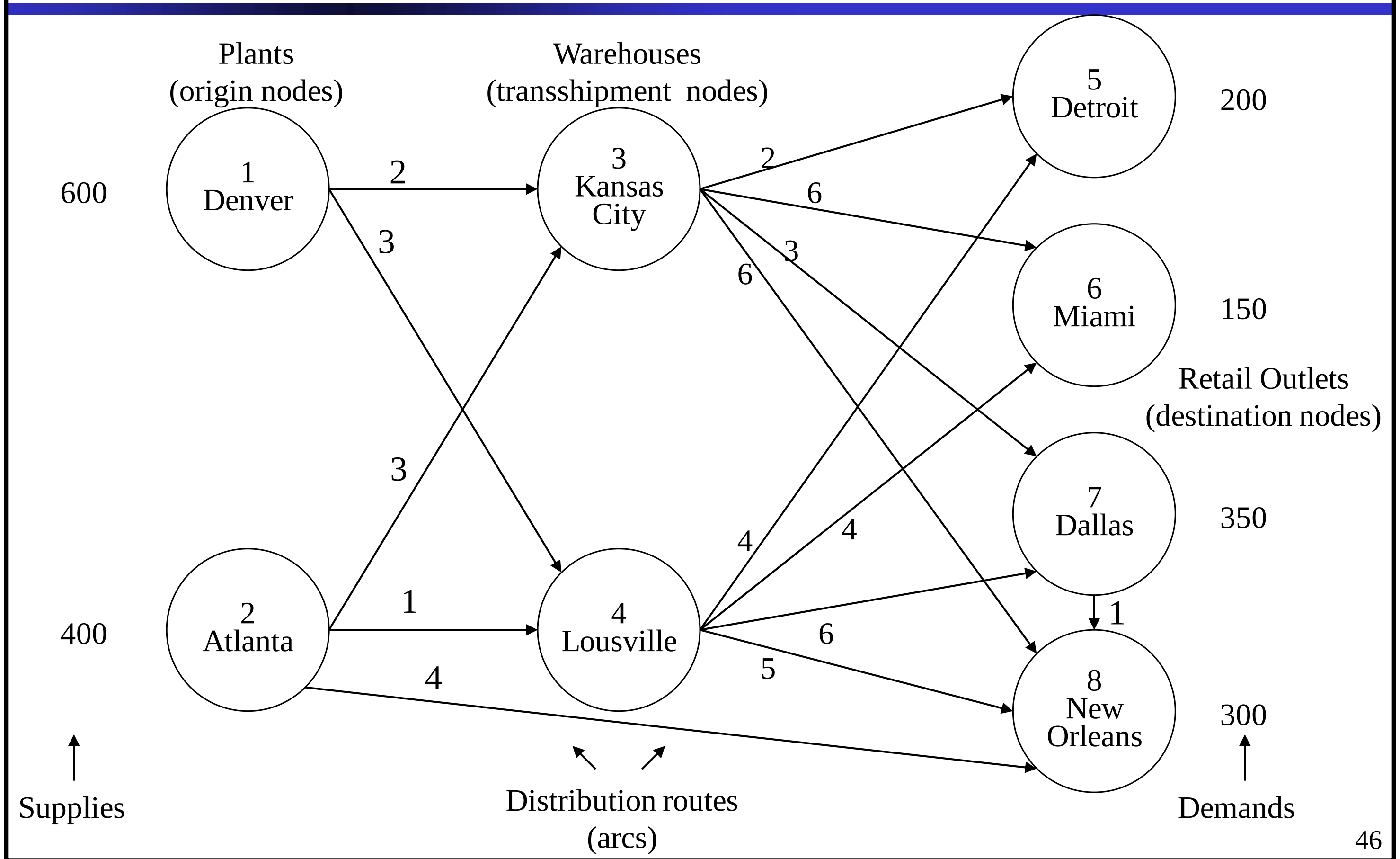
```
Objective Function Value =        5200.000

     Variable            Value            Reduced Costs
  --------------    ---------------    ------------------
       X13              550.000               0.000
       X14               50.000               0.000
       X23                0.000               3.000
       X24              400.000               0.000
       X35              200.000               0.000
       X36                0.000               1.000
       X37              350.000               0.000
       X38                0.000               0.000
       X45                0.000               3.000
       X46              150.000               0.000
       X47                0.000               4.000
       X48              300.000               0.000
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

Route				
From	**To**	**Units Shipped**	**Cost Per-Unit**	**Total Cost**
Denver	Kansas City	550	$2	$1100
Denver	Louisville	50	$3	150
Atlanta	Louisville	400	$1	400
Kansas City	Detroit	200	$2	400
Kansas City	Dallas	350	$3	1050
Louisville	Miami	150	$4	600
Louisville	New Orleans	300	$5	1500
				$5200

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM



ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

$$\text{Min}\quad 2x_{13}+3x_{14}+3x_{23}+1x_{24}+2x_{35}+6x_{36}+3x_{37}+6x_{38}+4x_{45}+4x_{46}+6x_{47}+5x_{48}+4x_{28}+1x_{78}$$

s.t.

$$\left.\begin{array}{l} x_{13}+x_{14} \leq 600 \\ x_{23}+x_{24}+x_{28} \leq 400 \end{array}\right\} 1$$

$$\left.\begin{array}{l} -x_{13}-x_{23}+x_{35}+x_{36}+x_{37}+x_{38} = 0 \\ -x_{14}-x_{24}+x_{45}+x_{46}+x_{47}+x_{48} = 0 \end{array}\right\} 2$$

$$\left.\begin{array}{l} x_{35}+x_{45} = 200 \\ x_{36}+x_{46} = 150 \\ x_{37}+x_{47}-x_{78} = 350 \\ x_{38}+x_{48}+x_{28}+x_{78} = 300 \end{array}\right\} 3$$

$x_{ij} \geq 0$ for all i and j

1. Origin node constraints
2. Transshipment node constraints
3. Destination node constraints

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

```
Objective Function Value =        4600.000

      Variable              Value              Reduced Costs
   --------------      ---------------      ------------------
         X13                600.000                   0.000
         X14                  0.000                   0.000
         X23                  0.000                   3.000
         X24                150.000                   0.000
         X35                200.000                   0.000
         X36                  0.000                   1.000
         X37                400.000                   0.000
         X38                  0.000                   2.000
         X45                  0.000                   3.000
         X46                150.000                   0.000
         X47                  0.000                   4.000
         X48                  0.000                   2.000
         X28                250.000                   0.000
         X78                 50.000                   0.000
```

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

A general linear programming model of the transshipment problem is

$$\text{Min} \sum_{\text{all arcs}} c_{ij} x_{ij}$$

s.t.

$$\sum_{\text{arcs out}} x_{ij} - \sum_{\text{arcs in}} x_{ij} \leq s_i \qquad \text{Origin nodes } i$$

$$\sum_{\text{arcs out}} x_{ij} - \sum_{\text{arcs in}} x_{ij} = 0 \qquad \text{Transshipment nodes}$$

$$\sum_{\text{arcs in}} x_{ij} - \sum_{\text{arcs out}} x_{ij} = d_j \qquad \text{Destination nodes } j$$

$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j$$

c_{ij} = per-unit cost of shipping from node i to node j

s_i = supply at origin i

d_j = demand at destination j

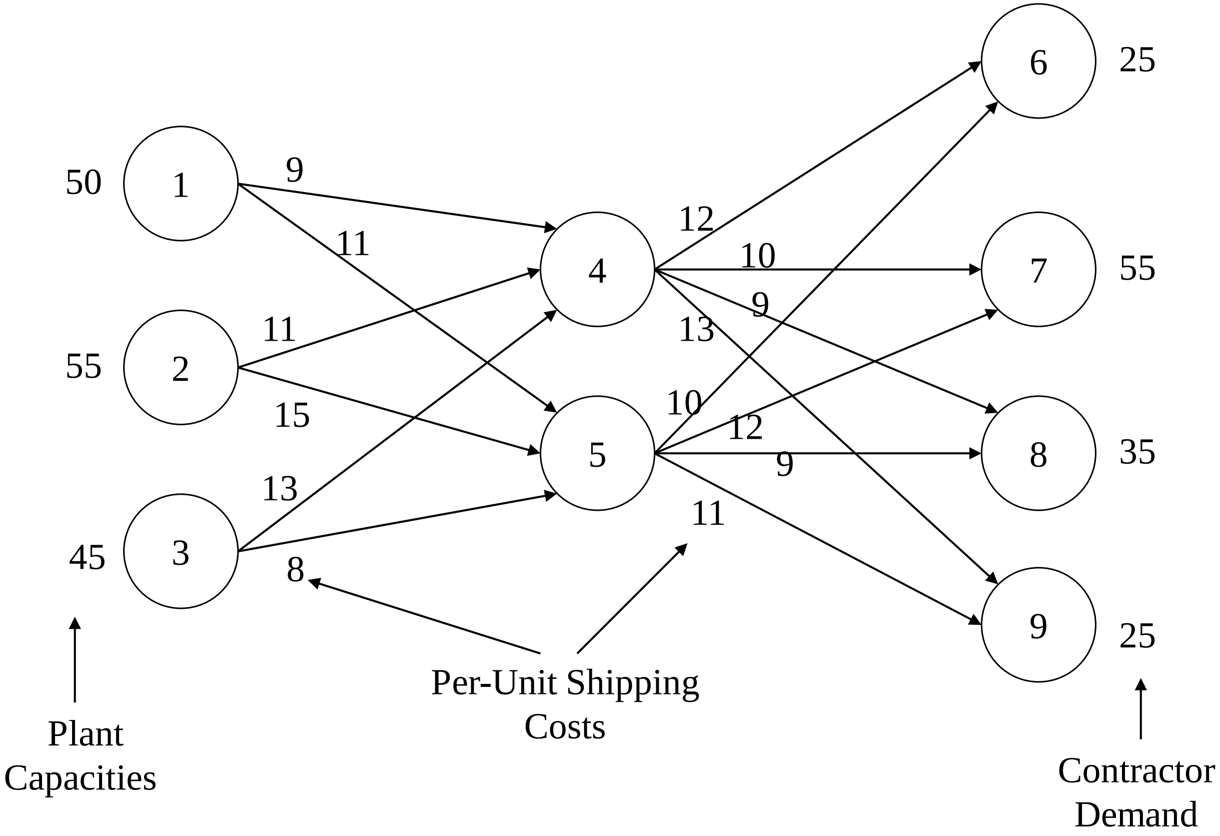
ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

Problem Description

Five Star Manufacturing Company makes compressors for air conditioners. The compressors are produced in 3 plants, then shipped on to 4 heating, ventilation, and air conditioning (HVAC) contractors.

A network model is shown below. Develop a linear programming model that Five Star can solve to minimize the cost of shipping compressors from the plants through the warehouses and on to the HVAC contractors.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM



Plant Capacities

Per-Unit Shipping Costs

Contractor Demand

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΤΩΣΗΣ
TRANSSHIPMENT PROBLEM

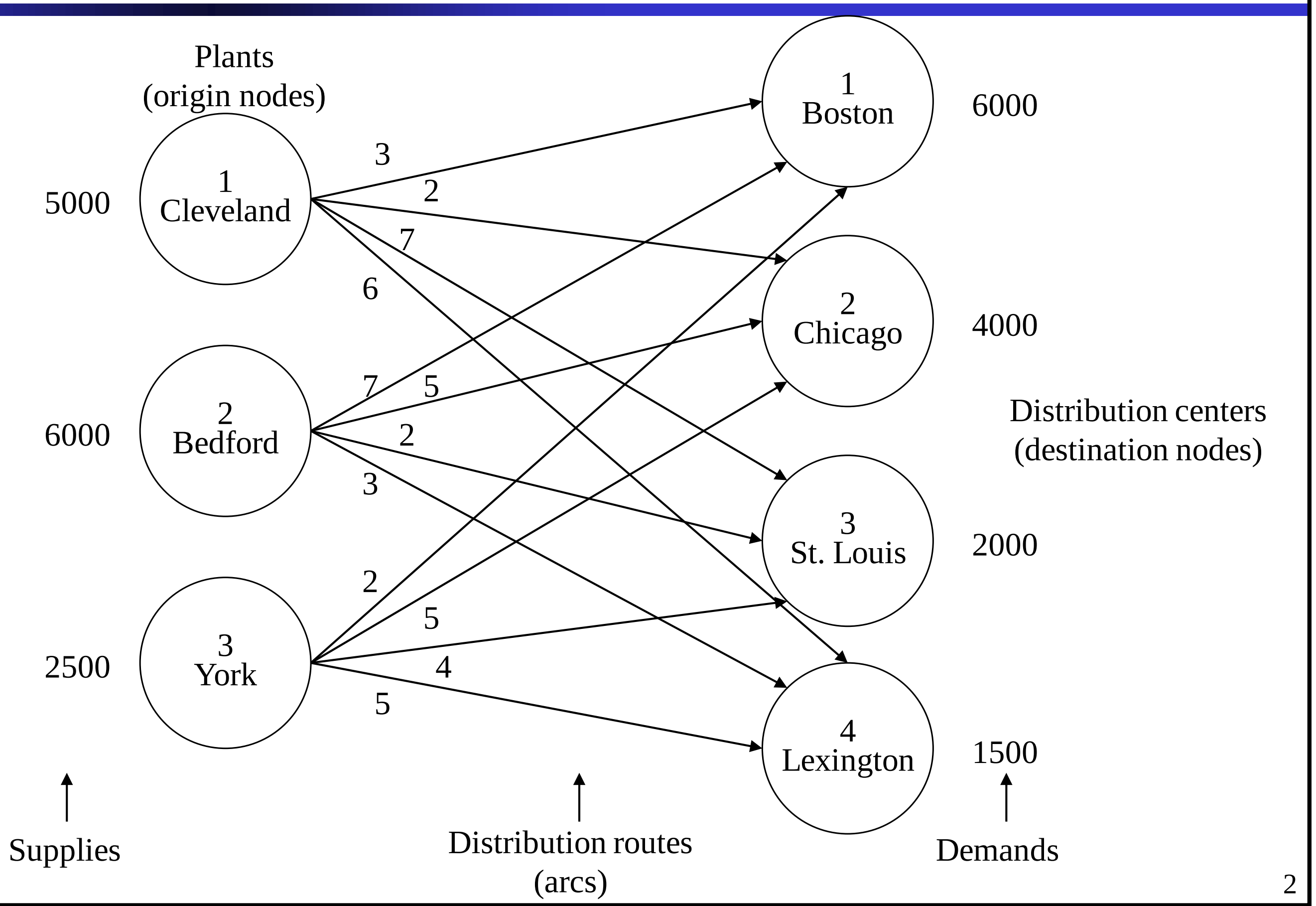
$$\begin{aligned}
\min \quad & 9x_{14} + 11x_{15} + 11x_{24} + 15x_{25} + 13x_{34} + 8x_{35} + 12x_{46} + 10x_{47} + 9x_{48} \\
& + 13x_{49} + 10x_{56} + 12x_{57} + 9x_{58} + 11x_{59}
\end{aligned}$$

s.t.

$$\begin{array}{lll}
x_{14} + x_{15} & \leq 50 & \text{Node 1} \\
x_{24} + x_{25} & \leq 55 & \text{Node 2} \\
x_{34} + x_{35} & \leq 45 & \text{Node 3} \\
-x_{14} - x_{24} - x_{34} + x_{46} + x_{47} + x_{48} + x_{49} & = 0 & \text{Node 4} \\
-x_{15} - x_{25} - x_{35} + x_{56} + x_{57} + x_{58} + x_{59} & = 0 & \text{Node 5} \\
x_{46} + x_{56} & = 25 & \text{Node 6} \\
x_{47} + x_{57} & = 55 & \text{Node 7} \\
x_{48} + x_{58} & = 35 & \text{Node 8} \\
x_{49} + x_{59} & = 25 & \text{Node 9}
\end{array}$$

$x_{ij} \geq 0$ for all i, j

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM



ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Πρώτη φάση : Εξεύρεση Αρχικής Εφικτής Λύσης

Phase I : Finding an Initial Feasible Solution

Πίνακας Μεταφοράς (Transportation Table)

	Destination				
Origin	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3	2	7	6	5,000
Bedford	7	5	2	3	6,000
York	2	5	4	5	2,500
Destination Demand	6000	4000	2000	1500	13500

Cell corresponding to shipments from Bedford to Boston

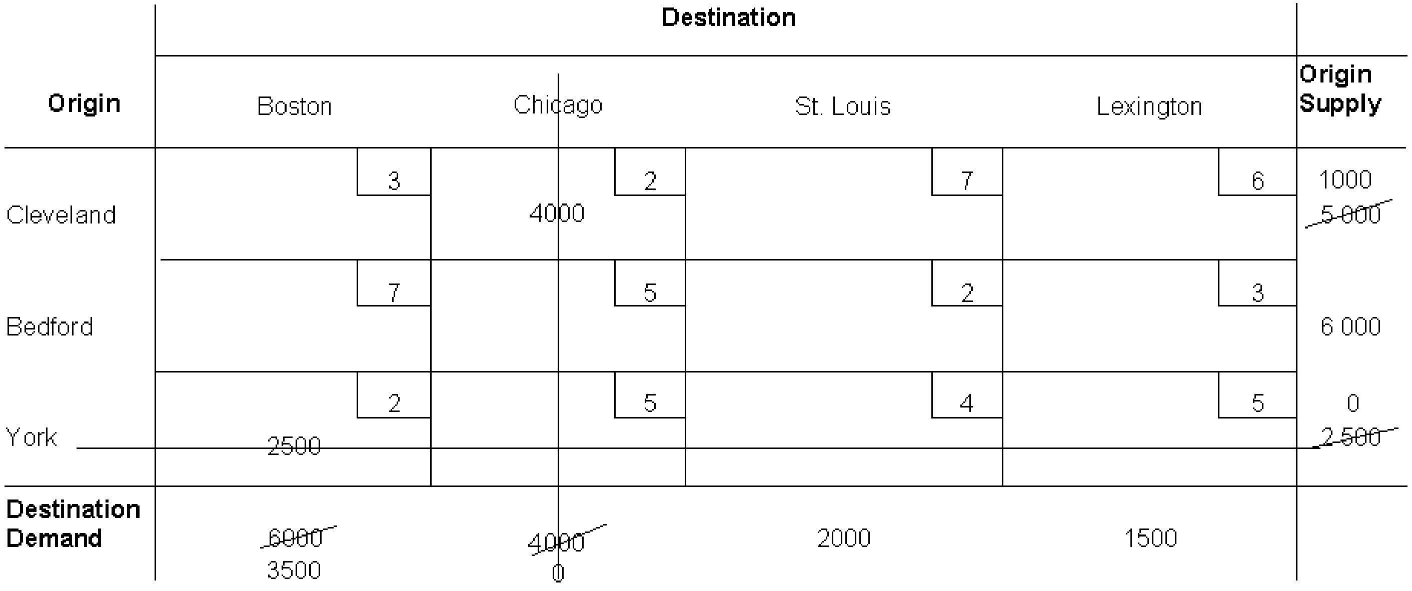
Total supply and total demand

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Μέθοδος Ελάχιστου Κόστους (Minimum Cost Method)

	Destination				
Origin	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3	2 4000	7	6	1000 ~~5 000~~
Bedford	7	5	2	3	6 000
York	2	5	4	5	2 500
Destination Demand	6000	4000 0	2000	1500	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM



	Destination				
Origin	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3	2 4000	7	6	1000 ~~5 000~~
Bedford	7	5	2	3	6 000
York	2 2500	5	4	5	0 ~~2 500~~
Destination Demand	~~6000~~ 3500	~~4000~~ 0	2000	1500	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Origin	**Destination**				**Origin Supply**
	Boston	Chicago	St. Louis	Lexington	
Cleveland	3	2 4000	7	6	1000 ~~5 000~~
Bedford	7	5	2 2000	3	4000 ~~6 000~~
York	2 2500	5	4	5	0 ~~2 500~~
Destination Demand	~~6000~~ 3500	~~4000~~ 0	~~2000~~ 0	1500	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

	Destination				
Origin	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3	2 4000	7	6	1000 ~~5 000~~
Bedford	7	5	2 2000	3 1500	2500 ~~4000~~ ~~6 000~~
York	2 ~~2500~~	5	4	5	0 ~~2 500~~
Destination Demand	~~6000~~ 3500	~~4000~~ 0	~~2000~~ 0	~~1500~~ 0	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

	Destination				
Origin	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3 1000	2 4000	7	6	0 ~~1000~~ ~~5 000~~
Bedford	7	5	2 2000	3 1500	2500 ~~4000~~ ~~6 000~~
York	2 2500	5	4	5	0 ~~2 500~~
Destination Demand	~~6000~~ ~~3500~~ 2500	~~4000~~ 0	~~2000~~ 0	~~1500~~ 0	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Origin	Destination				
	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3 1000	2 4000	7	6	0 ~~1000~~ ~~5 000~~
Bedford	7 2500	5	2 2000	3 1500	0 ~~2500~~ ~~4000~~ ~~6 000~~
York	2 2500	5	4	5	0 ~~2 500~~
Destination Demand	~~6000~~ ~~3500~~ ~~2500~~ 0	~~4000~~ 0	~~2000~~ 0	~~1500~~ 0	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Κόστος αρχικής Λύσης

Route		Units Shipped	Cost Per-Unit	Total Cost
From	To			
Cleveland	Boston	1000	$3	$3000
Cleveland	Chicago	4000	$2	$8000
Bedford	Boston	2500	$7	$17500
Bedford	St. Louis	2000	$2	$4000
Bedford	Lexington	1500	$3	$4500
York	Boston	2500	$2	$5000
				$42000

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Μέθοδος Ελάχιστου Κόστους

Step 1: Identify the cell in the transportation tableau with the lowest cost and allocate as much as possible to this cell. In case of a tie, choose the cell corresponding to the arc over which the most units can be shipped. If ties still exist, choose any of the tied cells.

Step 2: Reduce the row supply and the column demand by the amount of flow allocated to the cell identified in step 1.

Step 3: If all row supplies and column demands have been exhausted, then stop; the allocations made will provide an initial feasible solution. Otherwise, continue with step 4.

Step 4: If the row supply is now zero, eliminate the row from further consideration by drawing a line through it. If the column demand is now zero, eliminate the column by drawing a line through it.

Step 5: Continue with step 1 for all unlined rows and columns.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

- Κάθε κελί (cell) στον πίνακα μεταφοράς αντιστοιχεί με ένα τόξο στο δίκτυο.

- Προσπάθειά μας είναι να αναγνωρίσουμε ένα εισερχόμενο τόξο (incoming arc) και ένα εξερχόμενο τόξο (outgoing arc).

- Η μέθοδος MODI μας βοηθά στον πιο πάνω υπολογισμό.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Μέθοδος MODI:

u_i : δείκτης σειράς i

V_j : δείκτης στήλης j

Για κάθε κελλί που έχει καταληφθεί ισχύει το ακόλουθο:

$$C_{ij} = u_i + v_j$$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

	Destination				
Origin	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3 1000	2 4000	7	6	5000
Bedford	7 2500	5	2 2000	3 1500	6000
York	2 2500	5	4	5	2500
Destination Demand	6000	4000	2000	1500	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Occupied Cell	$u_i + v_j = c_{ij}$
Cleveland – Boston	$u_1 + v_1 = 3$
Cleveland – Chicago	$u_1 + v_2 = 2$
Bedford – Boston	$u_2 + v_1 = 7$
Bedford – St. Louis	$u_2 + v_3 = 2$
Bedford – Lexington	$u_2 + v_4 = 3$
York - Boston	$u_3 + v_1 = 2$

Θέτουμε $u_1 = 0$ τότε:

$0 + v_1 = 3$ $\quad u_2 + v_3 = 2$ $\quad\Longrightarrow\quad u_1 = 0$ $\quad v_1 = 3$

$0 + v_2 = 2$ $\quad u_2 + v_4 = 3$ $\quad u_2 = 4$ $\quad v_2 = 2$

$u_2 + v_1 = 7$ $\quad u_3 + v_1 = 2$ $\quad u_3 = -1$ $\quad v_3 = -2$

$v_4 = -1$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Για κάθε κελλί που δεν έχει καταληφθεί ισχύει:

$$\mathbf{e_{ij} = c_{ij} - u_i - v_j}$$

Δείκτης καθαρής αξιολόγησης

(net evaluation index)

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

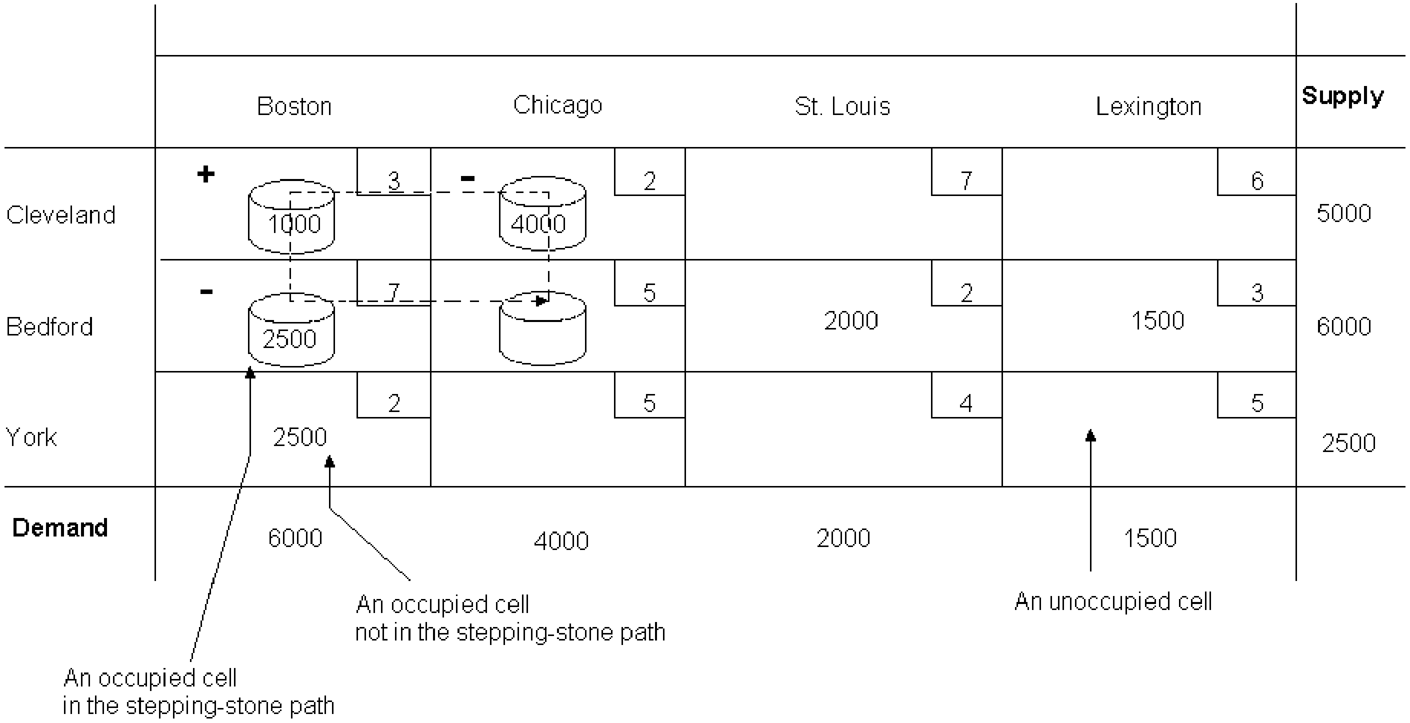
	v_j			
u_i	3	2	-2	-1
0	3 1000	2 4000	7 ⑨	6 ⑦
4	7 2500	5 (-1)	2 2000	3 1500
-1	2 2500	5 ④	4 ⑦	5 ⑦

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

	Destination				
Origin	Boston	Chicago	St. Louis	Lexington	**Origin Supply**
Cleveland	3 1001 ~~1000~~	2 3999 ~~4000~~	7	6	5000
Bedford	7 2499 ~~2500~~	5 1	2 2000	3 1500	6000
York	2 2500	5	4	5	2500
Destination Demand	6000	4000	2000	1500	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Stepping-Stone Method



	Boston	Chicago	St. Louis	Lexington	**Supply**
Cleveland	+ 3 1000	- 2 4000	7	6	5000
Bedford	- 7 2500	5	2 2000	3 1500	6000
York	2 2500	5	4	5	2500
Demand	6000	4000	2000	1500	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

	v_1	v_2	v_3	v_4	
	Boston	Chicago	St. Louis	Lexington	**Supply**
u_1 Cleveland	3 3500	2 1500	7	6	5000
u_2 Bedford	7	5 2500	2 2000	3 1500	6000
u_3 York	2 2500	5	4	5	2500
Demand	6000	4000	2000	1500	

$u_1 + v_1 = 3$ $u_1 + v_2 = 2$

$u_2 + v_1 = 7$ $u_2 + v_3 = 2$

$u_2 + v_4 = 3$ $u_3 + v_1 = 2$

Θέτουμε $u_1 = 0$ $\Longrightarrow$

$v_1 = 3$ $u_2 = 3$

$v_2 = 2$ $u_3 = -1$

$v_3 = -1$

$v_4 = 0$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

u_i	v_j			
	3	2	-1	0
0	3 3500	2 1500	7 ⑧	6 ⑥
3	7 ①	5 2500	2 2000	3 1500
-1	2 2500	5 ④	4 ⑥	5 ⑥

Route		Units Shipped	Cost Per-Unit	Total Cost
From	To			
Cleveland	Boston	3500	$3	$10500
Cleveland	Chicago	1500	$2	$3000
Bedford	Chicago	2500	$5	$12500
Bedford	St. Louis	2000	$2	$4000
Bedford	Lexington	1500	$3	$4500
York	Boston	2500	$2	$5000
				$39500

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

m: αριθμός πηγών

n: αριθμός προορισμών

- Εάν στον πίνακα μεταφοράς λιγότερα των (m + n –1) κελλιών έχουν καταληφθεί η λύση είναι εκφυλισμένη (degenerate solution).

-Εάν το πιο πάνω ισχύει, η μέθοδος MODI συναντά πρόβλημα στη χρήση της / λειτουργία της.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

u_i \ v_j	3	6		Supply
0	3 35	6 25	7	60
-1	8	5	7	30
	4	9	11 30	30
Demand	35	55	30	

$u_1 + v_1 = 3$

$u_1 + v_2 = 6$

$u_2 + v_2 = 5$

$u_3 + v_3 = 11$

Εάν $u_1 = 0$ →

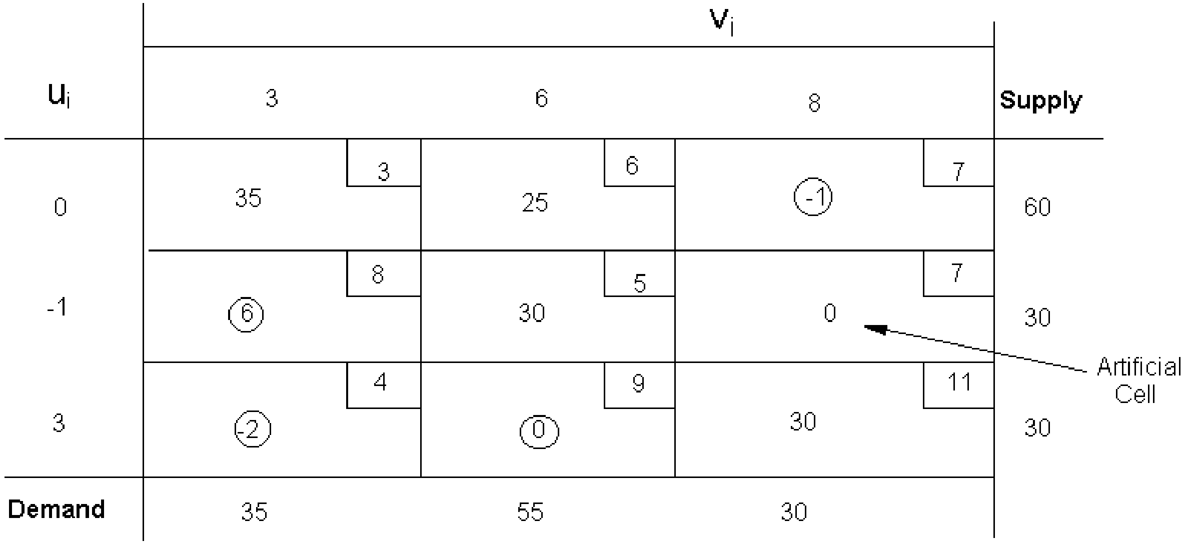
$v_1 = 3$

$v_2 = 6$

$u_2 = -1$

$u_3 = ?$, $v_3 = ?$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM



	v_i			
u_i	3	6	8	**Supply**
0	3 35	6 25	7 (-1)	60
-1	8 (6)	5 30	7 0	30
3	4 (-2)	9 (0)	11 30	30
Demand	35	55	30	

$u_1 + v_1 = 3$

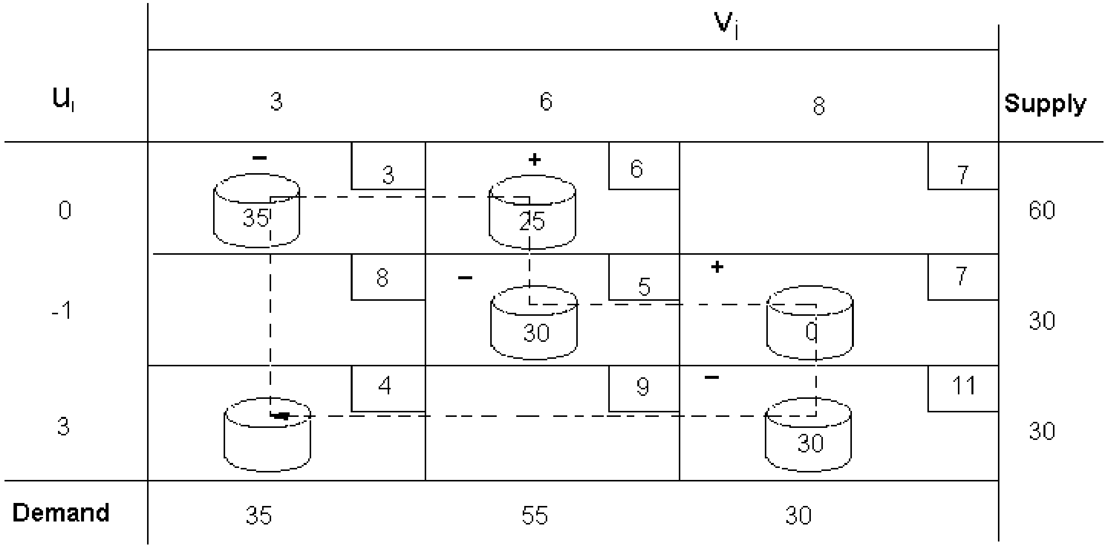
$u_1 + v_2 = 6$

$u_2 + v_2 = 5$

$u_2 + v_3 = 7$

$u_3 + v_3 = 11$

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

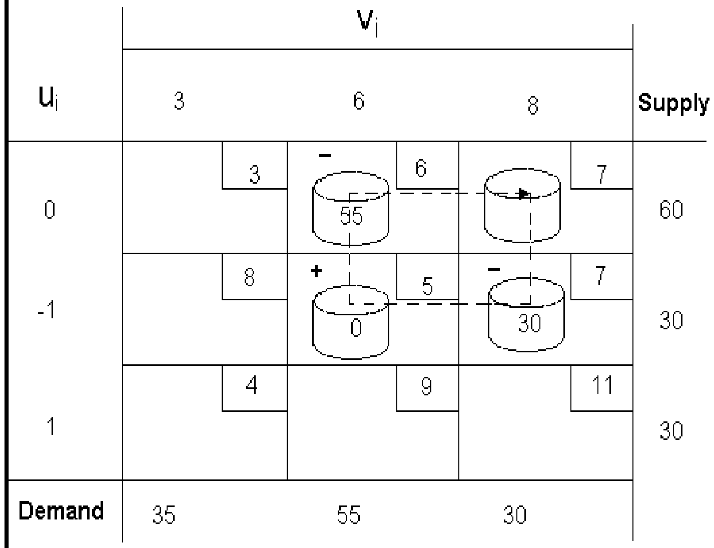


u_i	v_j 3	6	8	**Supply**
0	3 − 35	6 + 25	7	60
-1	8	5 − 30	7 + 0	30
3	4	9	11 − 30	30
Demand	35	55	30	

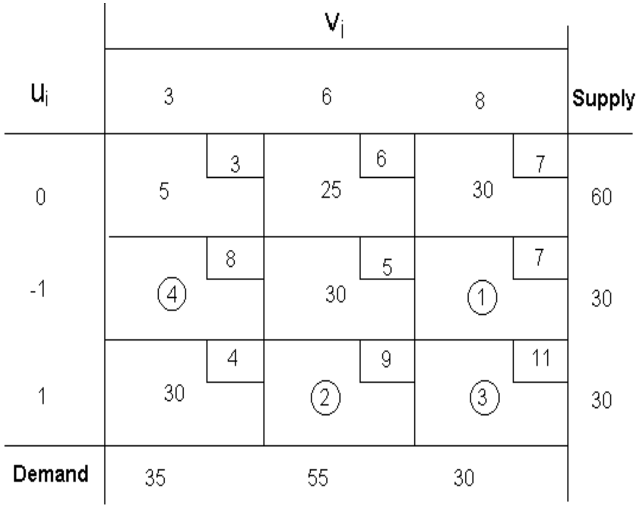
ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

	v_i			
u_i	3	6	8	**Supply**
0	3 5	6 55	7 (-1)	60
-1	8 ⑥	5 0	7 30	30
1	4 30	9 ②	11 ②	30
Demand	35	55	30	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM



u_i \ v_j	3	6	8	Supply
0	3	− 55 6	+ 7	60
-1	8	+ 0 5	− 30 7	30
1	4	9	11	30
Demand	35	55	30	



u_i \ v_j	3	6	8	Supply
0	5 3	25 6	30 7	60
-1	④ 8	30 5	① 7	30
1	30 4	② 9	③ 11	30
Demand	35	55	30	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

PHASE I

Find an initial feasible solution using the minimum cost method.

PHASE II

Step 1: If the initial feasible solution is degenerate, add an artificially occupied cell wherever necessary. Then use the MODI method to find u_i, v_j and identify the incoming cell. The incoming cell is the one with the smallest net evaluation index: $c_{ij} – u_i – v_j$. If $c_{ij} – u_i – v_j >=0$ for all unoccupied cells, stop; in this case, the optimal solution has been found. Otherwise, go to step 2.

Step 2: Find the stepping-stone path associated with the incoming cell. Label each cell on the stepping-stone path whose flow will increase with a plus sign and each cell whose flow will decrease with a minus sign.

Step 3: Choose as the outgoing cell the minus-sign cell on the stepping-stone path with the smallest flow. If there is a tie, choose any one of the tied cells. The tied cells that are not chosen will be artificially occupied with a flow of zero at the next iteration.

Step 4: Allocate to the incoming cell the amount of flow currently given to the outgoing cell; make the appropriate adjustments to all cells on the stepping-stone path, and continue with step 1.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Παραλλαγές Προβλήματος (Problem Variations)

- Εάν η συνολική προσφορά είναι μεγαλύτερη της ζήτησης, εισάγουμε εικονικό (dummy) προορισμό του οποίου η ζήτηση είναι ίση με τη διαφορά της συνολικής προσφοράς και ζήτησης.

- Εάν η συνολική ζήτηση είναι μεγαλύτερη της προσφοράς, εισάγουμε εικονική πηγή (origin) της οποίας η δυναμικότητα (capacity) είναι ίση με τη διαφορά της συνολικής ζήτησης και προσφοράς.

- Και στις δύο περιπτώσεις το κόστος μεταφοράς είναι μηδέν στη νέα τάξη.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Παραλλαγές Προβλήματος (Problem Variations)

-Για προβλήματα μεγιστοποίησης:

-Η δημιουργία της αρχικής εφικτής λύσης πραγματοποιείται με τη μέθοδο του μέγιστου κέρδους.

-Χρησιμοποιούμε τη μέγιστη τιμή e_{ij} για να βρούμε το εισερχόμενο τόξο.

- Μη αποδεκτές διαδρομές / τόξα (unacceptable arcs / routes)

1. **Πρόβλημα ελαχιστοποίησης**: Θέτουμε το κόστος μεταφοράς ίσο με M.
2. **Πρόβλημα μεγιστοποίησης**: Θέτουμε το κόστος μεταφοράς ίσο με -M.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Plants	Production Capacity
P_1	50
P_2	40
P_3	30
Total	120

Retail Outlets	Forecast Demand
R_1	45
R_2	15
R_3	30
Total	90

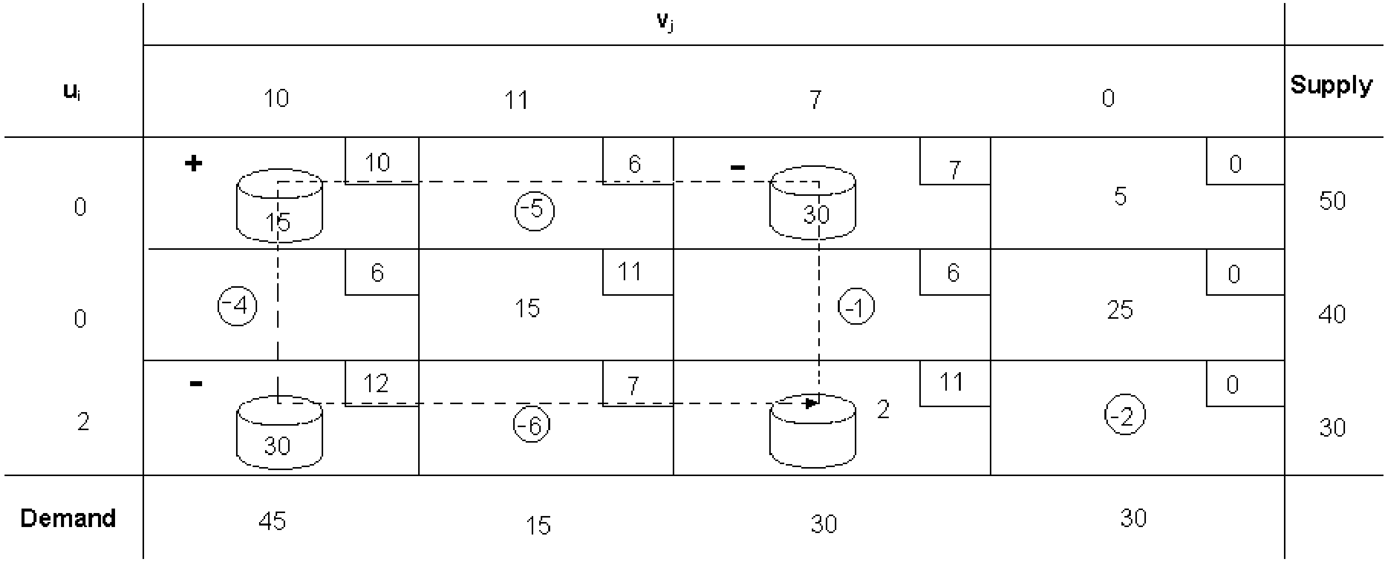
Plants	Retail Outlets		
	R_1	R_2	R_3
P_1	10	6	7
P_2	6	11	6
P_3	12	7	11

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Dummy Destination

	R_1	R_2	R_3	R_4	
P_1	10 15	6	7 30	0 5	50
P_2	6	11 15	6	0 25	40
P_3	12 30	7	11	0	30
	45	15	30	30	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM



	v_j				
u_i	10	11	7	0	**Supply**
0	+ 10 15	6 (-5)	- 7 30	0 5	50
0	6 (-4)	11 15	6 (-1)	0 25	40
2	- 12 30	7 (-6)	11 2	0 (-2)	30
Demand	45	15	30	30	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

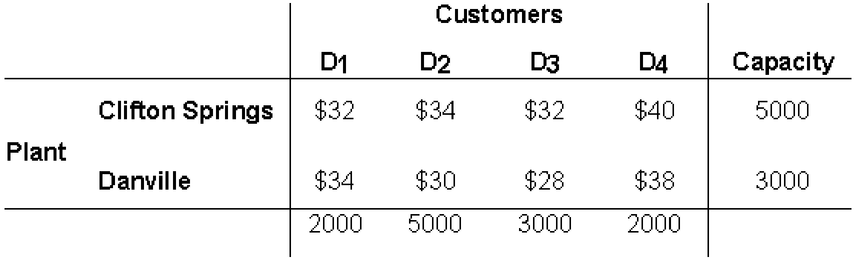
	v_j				
u_i	10	11	7	0	**Supply**
0	$\boxed{10}$ 45	$\boxed{6}$ $\textcircled{-5}$	$\boxed{7}$ 0	$\boxed{0}$ 5	50
0	$\boxed{6}$ $\textcircled{-4}$	$\boxed{11}$ 15	$\boxed{6}$ $\textcircled{-1}$	$\boxed{0}$ 25	40
2	$\boxed{12}$ $\textcircled{-2}$	$\boxed{7}$ $\textcircled{-8}$	$\boxed{11}$ 30	$\boxed{0}$ $\textcircled{-4}$	30
Demand	45	15	30	30	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Problem Description

Klein Chemicals, Inc. manufactures a product at two plants (Clifton Springs and Danville) and ships it to four different customers denoted by D_1, D_2, D_3, D_4).

The profit per unit for shipping from each plant to each customer, the order size or demand for each customer, and the plant capacities are shown below:



		Customers				
		D_1	D_2	D_3	D_4	**Capacity**
Plant	**Clifton Springs**	$32	$34	$32	$40	5000
	Danville	$34	$30	$28	$38	3000
		2000	5000	3000	2000	

Since the number of units ordered exceeds the plant capacities, we must add a dummy plant with a capacity of 4000 units.

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

The complete Transportation Tableau is:

		Customer				
		D_1	**D_2**	**D_3**	**D_4**	**Capacity**
Plant	Clifton Springs	32	34	32	40	5000
	Danville	34	30	28	38	3000
	Dummy	0	0	0	0	4000
	Demand	2000	5000	3000	2000	12000

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ

TRANSPORTATION PROBLEM

Initial Feasible Solution – Min Cost Method

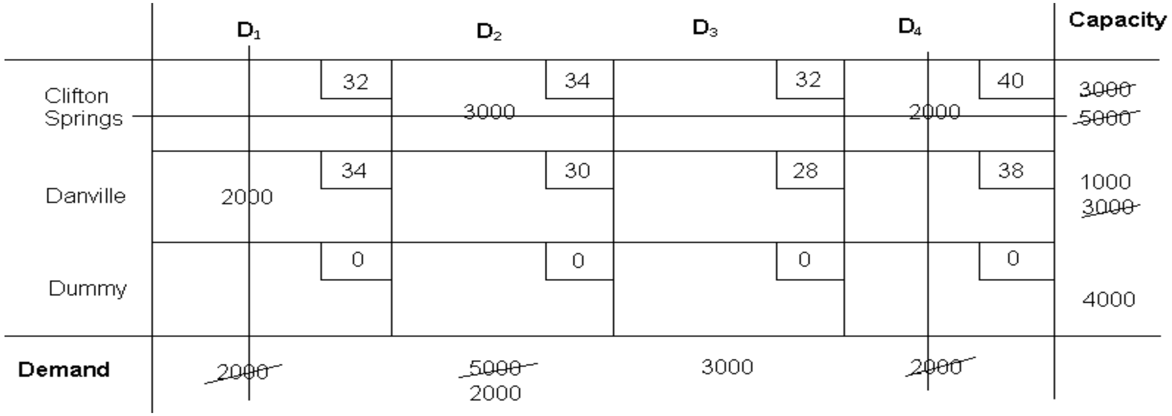
Note: In this case it is the Max Profit Method

	D₁	D₂	D₃	D₄	Capacity
Clifton Springs	32	34	32	40 2000	3000 ~~5000~~
Danville	34	30	28	38	3000
Dummy	0	0	0	0	4000
Demand	2000	5000	3000	~~2000~~	

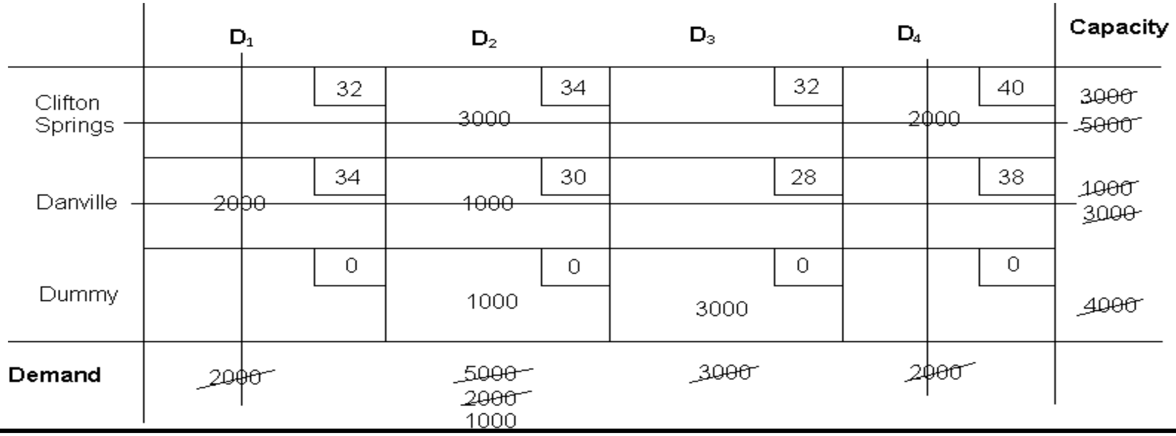
	D₁	D₂	D₃	D₄	Capacity
Clifton Springs	32	34 3000	32	40 2000	~~3000~~ ~~5000~~
Danville	34	30	28	38	3000
Dummy	0	0	0	0	4000
Demand	2000	~~5000~~ 2000	3000	~~2000~~	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Initial Feasible Solution Continued



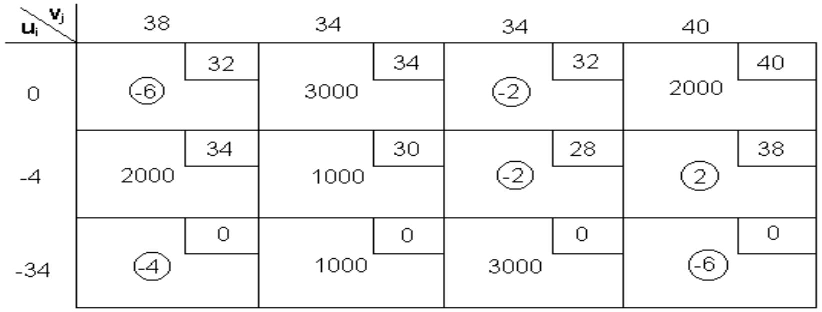
	D1	D2	D3	D4	Capacity
Clifton Springs	32	34 3000	32	40 2000	~~3000~~ ~~5000~~
Danville	34 2000	30	28	38	1000 ~~3000~~
Dummy	0	0	0	0	4000
Demand	~~2000~~	~~5000~~ 2000	3000	~~2000~~	



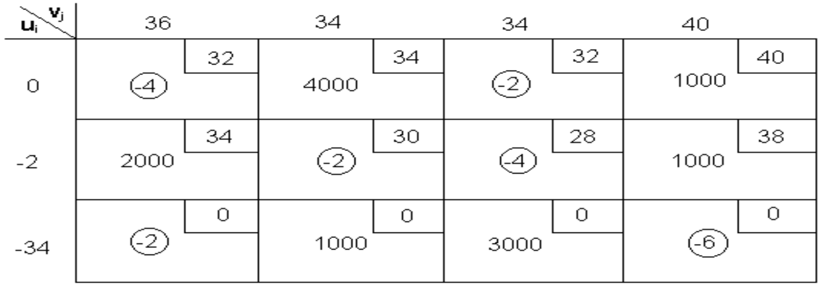
	D1	D2	D3	D4	Capacity
Clifton Springs	32	34 3000	32	40 2000	~~3000~~ ~~5000~~
Danville	34 2000	30 1000	28	38	~~1000~~ ~~3000~~
Dummy	0	0 1000	0 3000	0	~~4000~~
Demand	~~2000~~	~~5000~~ ~~2000~~ 1000	~~3000~~	~~2000~~	

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Apply MODI Method to find Optimal Solution:



u_i \ v_j	38	34	34	40
0	32 (-6)	34 3000	32 (-2)	40 2000
-4	34 2000	30 1000	28 (-2)	38 (2)
-34	0 (-4)	0 1000	0 3000	0 (-6)



u_i \ v_j	36	34	34	40
0	32 (-4)	34 4000	32 (-2)	40 1000
-2	34 2000	30 (-2)	28 (-4)	38 1000
-34	0 (-2)	0 1000	0 3000	0 (-6)

Since the per unit change for each new cell is less than or equal to zero, we have reached the optimal solution

ΠΡΟΒΛΗΜΑ ΜΕΤΑΦΟΡΑΣ
TRANSPORTATION PROBLEM

Optimal Solution:

Route	Units	Profit/ Unit	Profit
Clifton Springs to D_2	4000	\$34	\$136,000
Clifton Springs to D_4	1000	40	40,000
Danville to D_1	2000	34	68,000
Danville to D_4	1000	38	38,000
Danville to D_2	1000	0	0
Danville to D_3	3000	0	0

Total Profit = \$282,000

Each plant should produce at its capacity

Customer D_2 is not satisfied (1000 units short)

Customer D_3 is not satisfied (3000 units short)

ΔΙΟΙΚΗΤΙΚΗ ΕΠΙΣΤΗΜΗ
ΔΔΕ 241

ΓΙΩΡΓΟΣ ΧΑΤΖΗΝΙΚΟΛΑΣ

ΕΝΟΤΗΤΑ 6:

ΠΡΟΒΛΗΜΑΤΑ ΜΕΤΑΦΟΡΑΣ, ΕΚΧΩΡΗΣΗΣ ΚΑΙ ΜΕΤΑΦΟΡΤΩΣΗΣ

III

ΤΜΗΜΑ ΔΗΜΟΣΙΑΣ ΔΙΟΙΚΗΣΗΣ ΚΑΙ ΔΙΟΙΚΗΣΗΣ ΕΠΙΧΕΙΡΗΣΕΩΝ
ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΥΠΡΟΥ

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Hungarian Method

	Client		
Project Leader	1	2	3
Terry	10	15	9
Carle	9	18	5
McClymonds	6	14	3

Step 1: Reduce the initial matrix by subtracting the smallest element in each row from every element in that row. Then, using the row-reduced matrix, subtract the smallest element in each column from every element in that column.

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

	Client		
Project Leader	1	2	3
Terry	1	6	0
Carle	4	13	0
McClymonds	3	11	0

	Client		
Project Leader	1	2	3
Terry	0	0	0
Carle	3	7	0
McClymonds	2	5	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Step 2: Find the minimum number of straight lines that must be drawn through the rows and the columns of the current matrix so that all the zeros in the matrix will be covered. If the minimum number of straight lines is the same as the number of rows (or equivalently, columns) in the matrix, an optimal assignment with a value of zero can be made. If the minimum number of straight lines is less than the number of rows, go to step 3.

	Client		
Project Leader	1	2	3
Terry	0	0	0
Carle	3	7	0
McClymonds	2	5	0

Two straight lines will cover all the zeros (step 2)

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Step 3: Subtract the value of the smallest unlined element from every unlined element, and add this same value to every element at the intersection of two lines. All others elements remain unchanged. Return to step 2, and continue until the minimum number of lines necessary to cover all the zeros in the matrix is equal to the number of rows.

	Client		
Project Leader	1	2	3
Terry	0	0	2
Carle	1	5	0
McClymonds	0	3	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

	Client		
Project Leader	1	2	3
Terry	~~0~~	~~0~~	~~2~~
Carle	1	5	~~0~~
McClymonds	~~0~~	~~3~~	~~0~~

Step 2

Three straight lines must be drawn to cover all zeros; therefore, the optimal solution has been reached

	Client		
Project Leader	1	2	3
Terry	0	$\boxed{0}$	2
Carle	1	5	$\boxed{0}$
McClymonds	$\boxed{0}$	3	0

Optimal Solution

Cost = 15 + 5 + 6 = 26

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Παραλλαγές προβλήματος

1) Εάν ο αριθμός Εργατών είναι > από αριθμό Εργασιών

   If the number of Agents > number of Clients

- Προσθέτουμε εικονική Εργασία (Dummy Client) με μηδενικό κόστος.

2) Εάν ο αριθμός Εργατών είναι < από αριθμό Εργασιών

   If the number of Agents < number of Clients

- Προσθέτουμε εικονικό Εργάτη (Dummy Agent) με μηδενικό κόστος.

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

	Client		
Project Leader	1	2	3
Terry	10	15	9
Carle	9	18	5
McClymonds	6	14	3
Higley	8	16	6

Dummy Client

		Client		
Project Leader	1	2	3	D
Terry	10	15	9	0
Carle	9	18	5	0
McClymonds	6	14	3	0
Higley	8	16	6	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

After adding a dummy column, we get an initial assignment matrix

	Client			
Project Leader	1	2	3	D
Terry	10	15	9	0
Carle	9	18	5	0
McClymonds	6	14	3	0
Higley	8	16	6	0

Applying steps 1 and 2 we obtain:

	Client			
Project Leader	1	2	3	D
Terry	4	①	6	~~0~~
Carle	3	4	2	~~0~~
McClymonds	~~0~~	~~0~~	~~0~~	~~0~~
Higley	2	2	3	~~0~~

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Applying step 3 followed by step 2 result in:

	Client			
Project Leader	1	2	3	D
Terry	3	0	5	0
Carle	2	3	1	0
McClymonds	0	0	0	1
Higley	1	1	2	0

Applying step 3 followed by step 2 result in:

	Client			
Project Leader	1	2	3	D
Terry	3	0	5	1
Carle	1	2	0	0
McClymonds	0	0	0	2
Higley	0	0	1	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Optimal solution

	Client			
Project Leader	1	2	3	D
Terry	3	0	5	1
Carle	1	2	0	0
McClymonds	0	0	0	2
Higley	0	0	1	0

Terry:	Client 2 (15 days)	
Carle:	Client 3 (5 days)	Total time = 26 days
McClymonds:	Client 1 (6 days)	
Higley:	Unassigned	

Note: An alternative optimal solution is:

Terry:	Client 2 (15 days)	
Carle:	Unassigned	Total time = 26 days
McClymonds:	Client 3 (3 days)	
Higley:	Client 1 (8 days)	

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Παραλλαγές προβλήματος: **Πρόβλημα Μεγιστοποίησης**

	Location			
Department	1	2	3	4
Shoe	10	6	12	8
Toy	15	18	5	11
Auto parts	17	10	13	16
Housewares	14	12	13	10
Video	14	16	6	12

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

			Location		
Department	1	2	3	4	5
Shoe	10	6	12	8	0
Toy	15	18	5	11	0
Auto parts	17	10	13	16	0
Housewares	14	12	13	10	0
Video	14	16	6	12	0

Dummy Location

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Μετατρέπουμε τον πίνακα σε **Opportunity losses**, αφαιρώντας κάθε στοιχείο από το μέγιστο στοιχείο της στήλης που ανήκει.

	Location				
Department	1	2	3	4	5
Shoe	7	12	1	8	0
Toy	2	0	8	5	0
Auto parts	0	8	0	0	0
Housewares	3	6	0	6	0
Video	3	2	7	4	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

We start with the opportunity loss matrix

	Location				
Department	1	2	3	4	5
Shoe	7	12	1	8	0
Toy	2	0	8	5	0
Auto parts	0	8	0	0	0
Housewares	3	6	0	6	0
Video	3	2	7	4	0

	Location				
Department	1	2	3	4	5
Shoe	5	12	1	6	0
Toy	0	0	8	3	0
Auto parts	0	10	2	0	2
Housewares	1	6	0	4	0
Video	1	2	7	2	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ

ASSIGNMENT PROBLEM

			Location		
Department	1	2	3	4	Dummy
Shoe	4	11	0	5	$\boxed{0}$
Toy	0	$\boxed{0}$	8	3	1
Auto parts	0	10	2	$\boxed{0}$	3
Housewares	1	6	$\boxed{0}$	4	1
Video	$\boxed{0}$	1	6	1	0

Optimal solution	Location	Profit
Shoe	Dummy	0
Toy	2	18
Auto parts	4	16
Housewares	3	13
Video	1	14
		61

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Παραλλαγές Προβλήματος: **Μη αποδεκτές Εκχωρήσεις / Unacceptable Assignments**

M for unacceptable minimization assignments

-M for unacceptable maximization assignments

	Location				
Department	1	2	3	4	Dummy
Shoe	10	6	12	8	0
Toy	15	-M	5	11	0
Auto parts	17	10	13	-M	0
Housewares	14	12	13	10	0
Video	14	16	6	12	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Subtracting each element from the largest element in its column leads to the **Opportunity Loss** matrix

	Location				
Department	1	2	3	4	Dummy
Shoe	7	10	1	4	0
Toy	2	M	8	1	0
Auto parts	0	6	0	M	0
Housewares	3	4	0	2	0
Video	3	0	7	0	0

	Location				
Department	1	2	3	4	Dummy
Shoe	6	9	1	3	0
Toy	1	M	8	0	0
Auto parts	0	6	1	M	1
Housewares	2	3	0	1	0
Video	3	0	8	0	1

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

			Location		
Department	1	2	3	4	Dummy
Shoe	6	9	1	3	$\boxed{0}$
Toy	1	M	8	$\boxed{0}$	0
Auto parts	$\boxed{0}$	6	1	M	1
Housewares	2	3	$\boxed{0}$	1	0
Video	3	$\boxed{0}$	8	0	1

Optimal solution	Location	Profit
Shoe	Dummy	0
Toy	4	11
Auto parts	1	17
Housewares	3	13
Video	2	16
		57

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Problem Description:

The Manager for a heating and air conditioning company has three jobs to send technicians on and four technicians available. She would like to assign technicians to the calls so that total service time is minimized.

	Job		
Techician	1	2	3
1	40	27	36
2	28	30	22
3	33	36	28
4	30	38	26

Add Dummy Job

Dummy Job

	Job			
Techician	1	2	3	Dummy
1	40	27	36	0
2	28	30	22	0
3	33	36	28	0
4	30	38	26	0

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

Row Reduction: Not necessary. Each row already has a zero since a dummy job was added.

Column Reduction:

		Job		
Techician	1	2	3	Dummy
1	12	0	14	0
2	0	3	0	0
3	5	9	6	0
4	2	11	4	0

After column reduction, the minimum unlined element is found to be 2

		Job		
Techician	1	2	3	Dummy
1	12	~~0~~	14	~~0~~
2	~~0~~	~~3~~	~~0~~	~~0~~
3	5	~~9~~	6	~~0~~
4	2	~~11~~	4	~~0~~

ΠΡΟΒΛΗΜΑ ΕΚΧΩΡΗΣΗΣ
ASSIGNMENT PROBLEM

	Job			
Techician	1	2	3	Dummy
1	10	$\boxed{0}$	12	0
2	0	5	$\boxed{0}$	2
3	3	9	4	$\boxed{0}$
4	$\boxed{0}$	11	2	0

Optimal solution	Job	Hours
1	2	27
2	3	22
3	Dummy	0
4	1	30
Total hours		79

Technician 3 is not assigned.